On the interpretation of the conceptual content of the state vector on the basis of the EPR– and GHZ–experiment

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Abstract

This paper will deal with the domain of questions related to the Heisenberg Uncertainty Relationship. It will be shown by a number of experiments, some merely gedankenexperiments, others completely experimentally verifiable — and indeed also verified —, that, more than as a practical limitation on the human scientific knowledge about the state of affairs of physical reality, the quantum mechanical predictions should be interpreted as implying the conclusion that the concept of property, as applied to physical entities such as particles, on the microscopic level cannot be considered well defined. The main reason for this conclusion is that it can be shown on the basis of the outcomes of the EPR– and GHZ–experiments, via Bell's Theorem, that a model which incorporates instruction sets to represent properties is impossible.

1 Introduction

1.1 Heisenberg Uncertainty Relationship

Heisenberg's Uncertainty Principle is often explained by an optical example, which, though pedagogically appropriately, only makes clear that the system will be disturbed as a result of the measurement and that it is thus impossible to know physical quantities such as the position or momentum of a particle to an arbitrary precision. Though illuminating in a computational context, the explanation does not allow any conclusion about the nature of position and momentum of a particle; — it could well be that momentum and position can both be well–defined for a particle, but that it is simply impossible to know them both at the same time. In this paper it will be held that this reasoning is essentially the wrong way around; rather it must be concluded first that objects do not have fixed properties, and, *consequently*, that the measurement, which nevertheless revealed a fixed property must have disturbed the system. The precise meaning of "fixed" properties will be elaborated on later.

Until later than halfway the 20th century this question was considered of philosophical nature and the answer not within the domain of physical experimentation. It was thus that Einstein, Podolsky and Rosen felt confident to include a positive answer to it as an assumption in their argument for the incompleteness of quantum mechanics. In 1964 John Bell published a paper that showed that their gedankenexperiment, instead of supporting their argument, showed the inconsistency between their assumptions and the predictions of quantum mechanics, and that these latter facts seemed to lead up to the conclusion that generally properties as we understand them on the macroscopic scale of the everyday life are simply not well–defined on a microscopic level.

1.2 Outline

The remainder of this article is organized as follows. Initially, in section 2, there will be a philosophical discussion to introduce the terminology that will be deployed in this paper in a broader context. The discussion proceeds in section 3 from a concrete and fundamentally simple physical system of the EPR experiment, as proposed by Einstein, Podolsky and Rosen in 1935, to the more general argument that David Mermin has given on the basis of an analogous experiment specified in generalised concepts but which shows the essential nature of the argument that Bell published in 1964 in conjunction with the Bell Inequalities is elaborated on in section 4. A similarly adapted version of Mermin's GHZ experiment is then given in section 5, the exact physical configuration of which will then be given in section 6. The physical predictions will be discussed, and how it can adjusted so that it precisely behaves as Mermin's model assumes. In section 7 there will be an overview of the argumentation and a connection made with the philosophical background discussed before.

2 Philosophical Fundamentals

2.1 The nature of properties

In epistemology — the philosophical study of knowledge — several definitions of the nature of properties have been postulated. In this section some of them will be briefly discussed.

The 16th century philosopher John Locke[3] defined properties (qualities) as powers to produce ideas in a mind. For instance, a ball can produce the idea of roundness in a mind through its being perceived, and thus the property roundness can be attributed to the ball.

Locke then distinguishes between primary and secondary qualities. Primary qualities are resemblances; they resemble something that is really there in the object. Locke gives the examples of solidity, extension and motion. Secondary qualities, however, are often thought to be resemblances, but actually are not. For instance, if we would first have had our hands in a cold environment, then a bucket of water of room temperature will feel warm to us, but if we would have been in a hot bath before, then the same bucket of water feels cold. According to Locke, this leads to the inevitable conclusion that temperature cannot be said to be present in the object. In Locke's view, these are merely the result of one — or the simultaneous effect of many — of the primary qualities. For instance, in the case of temperature, the motion of the water molecules would categorise as a primary quality, and its temperature, a secondary quality, would be (1) derived from these movements and (2) dependent on the observer.

Interestingly, since qualities are powers to produce ideas in the mind of an observer, Locke has defined qualities by the perceiver and not primarily by what is present in the object. This will be of importance to the discussion that is to follow, where, analogously, particles — prior to knowing the nature of the detector that reacts to its arrival — are ascribed qualities on the basis of this reaction only. For instance, if it would happen to be so that a certain detector could exist, that would flash a light marked "1" or "2" upon the arrival of a certain entity, whose exact nature does not need to be known, and if there is some consistency in this flashing pattern — for instance that one entity, every time it is subjected to the detector, results in the light "1" flashing — then that in itself constitutes a reason to attribute to that particular entity the quality of "1–ness," properly understood. That the semantic content of the concept of any property is only precisely this, will be an essential assumption of the discussion in this paper.

Also, had the flashing pattern been completely random, then there would be no inclination in us to ascribe any connection between the actions of the detector and the nature of the entity that it reacts to. A more subtle situation is achieved in the experiments that will be discussed here. Assume that there exists, hypothetically, a device that could still flash either of the two lights "1" or "2" upon the arrival of one and the same entity, but the ratio between the occurences of either of the lights varies as different entities are subjected to it. In this case we would probably be inclined to say that there is something in the particle, though it cannot be definite "1–ness" or "2–ness," that results in pattern in the flashing of the lights.

Finally, qualities can be said to be the ways in which objects present themselves to us. For instance, the perception of an apple, Locke holds, is the perception of a finite set of its attributes, such as its roundness, its softness and its taste. Upon perceiving all — and yet *only* — these attributes, it is often found that we are completely justified to postulate the existence of the object apple. Yet we have then not seen the object in itself; all that we have seen are its qualities, and there is no reason to assume that there must be some entity out there to which these qualities belong. This entity, which is postulated in the act of human cognition, Locke calls "substance" (appropriately from the terms "sub" — underneath, and "stance" — standing) and the idea we have of it results merely from our assumption that there must be something supporting the qualities that are perceived.

2.2 Esse est percipi

George Berkeley[1] starts from a similar definition of qualities but proceeds further, to assert that the being of this entity is uniquely its being perceived — that "esse est percipi." Essential characteristics of his argument will be incorporated in the discussion of this paper, and the explanation in this section is hoped to reveal that it is essentially less distant from common sense than would seem at first glance.

Berkeley anticipates and reacts to criticism in his book "A Treatise Concerning the Principles of Human Knowledge" (I,23)[1]:

> But, you say, surely there is nothing easier than to imagine trees, for instance, in a park or books existing in a closet and nobody nearby to perceive them. I answer: you may so, there is no difficulty in it; but what is all this, I beseech you, more than framing in your mind certain ideas which you call *books* and *trees* and at the same time omitting to frame the idea of anyone that may perceive them? But do not you yourself perceive or think of them all the while?"

The adaptation of Berkeley's argument as it will be deployed in this paper is captured in the statement that our human understanding cannot think of the physical world but in *terms of perception* — where "terms" not exclusively refers to the unit of language, but more generally to the unit of thought. Berkeley argued that even though we can imagine unobserved things as existing, yet whenever we do think about those things, all we do is imagining *seeing* them being somewhere where they cannot be perceived. But then, in our imagination, they are still being perceived. It will be held in this paper that it is impossible for the human intellect to conceive of things outside of their being perceived. On the basis of Locke's observation that substance cannot directly be perceived, but only its properties, this argument ultimately leads to the conclusion that properties uniquely constitute a fundamental unit of thought about nature.

However, it might seem that in quantum mechanics, with the postulation of the existence of a "state vector," it would have finally become possible to transcend the thinking in concrete, observed, properties and arrive at the concept of a particle not having fixed qualities, but rather a superposition of them, and it, only once observed assuming certain properties. The more fundamental question to be addressed in this paper is whether this attitude is correct, and the EPR– and GHZ–experiments will serve as an important element of the argumentation.

3 EPR Experiment

3.1 Preliminary Discussion of the Singlet State

The discussion in this section closely follows R. Shankar's treatment of spin singlet states[8].

First of all, we consider a system of two particles of spin magnitude $\frac{1}{2}$. The states are denoted by $|\pm_1 \pm_2\rangle$, where $\pm_i$ denotes spin up or down for the particle i ($i = 1, 2$). The basis of the four-dimensional product space of spin configurations of the entire system is then given by these kets $|\pm_1 \pm_2\rangle$.

Defined is the operator $S_z = S_{1z} + S_{2z}$, whose action on the basis kets is then:

$$S_z|\pm_1 \pm_2\rangle = (\pm\frac{\hbar}{2} \pm \frac{\hbar}{2})|\pm_1 \pm_2\rangle \tag{1}$$

Interestingly, the nullspace of S_z is given by $S_z|+_1 -_2\rangle = 0|+_1 -_2\rangle$ and $S_z|-_1 +_2\rangle = 0|-_1 +_2\rangle$. Thus to the eigenvalue $s_z = 0$ there corresponds a two–dimensional degenerate eigenspace spanned by the kets $|-_1 +_2\rangle$ and $|+_1 -_2\rangle$.

Defined is the operator $S^2 = (\mathbf{S}_1 + \mathbf{S}_2) \cdot (\mathbf{S}_1 + \mathbf{S}_2) = S_1^2 + S_2^2 + 2(\mathbf{S}_1 \cdot \mathbf{S}_2)$. Eigenkets are then $|+_1 +_2\rangle$ and $|-_1 -_2\rangle$ but not $|+_1 -_2\rangle$ and $|-_1 +_2\rangle$. However, their sum and difference are.

Thus the following kets are simultaneous eigenstates of both S_z and S^2:

$$|+_1 +_2\rangle \qquad s_z = 1 \quad s = 1 \tag{2}$$

$$\frac{1}{\sqrt{2}}(|+_1 -_2\rangle + |-_1 +_2\rangle) \qquad s_z = 0 \quad s = 1 \tag{3}$$

$$|-_1 -_2\rangle \qquad s_z = -1 \quad s = 1 \tag{4}$$

$$\frac{1}{\sqrt{2}}(|+_1 -_2\rangle - |-_1 +_2\rangle) \qquad s_z = 0 \quad s = 0 \tag{5}$$

Of interest to this discussion will only be the last of these four states (equation 5) and it will be assumed that a set of two particles can be produced that with certainty is in this *singlet* state.

3.2 Configuration

The experimental setup of the EPR experiment will now be discussed. Initially introduced as a gedankenexperiment, it has later been physically performed, although the argument can be understood as being meaningful even in the absence of this verification. The discussion in this section closely follows the discussion by Greenberger, Horne, Shimony and Zeilinger[2].

In the EPR experiment there are two particles, together in the singlet state of equation 5. Particle 1 is subjected to a spin measurement along a direction $\hat{\mathbf{a}}$ by a device such as a Stern–Gerlach apparatus, and particle 2 likewise along a direction $\hat{\mathbf{b}}$. The outcome of this measurement for particle 1 will be referred to as $A(\hat{\mathbf{a}}) = \pm 1$, with $+1$ for spin up and -1 for spin down, and likewise $B(\hat{\mathbf{b}}) = \pm 1$ for particle 2.

Two things are noteworthy about this particular state, which will from now on be abbreviated as

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|+-\rangle - |-+\rangle) \tag{6}$$

First of all there is what will be referred to as *perfect (anti)correlation*, which means that if the spin of one particle is measured along an arbitrary direction $\hat{\mathbf{n}}$, the outcome of a spin measurement of the other particle can, with certainty, be predicted to be opposite. More concretely, if the one particle is measured to have spin up along $\hat{\mathbf{n}}$, then the other particle will have spin down along $\hat{\mathbf{n}}$ and vice versa. Mathematically formulated, this is $A(\hat{\mathbf{n}}) = -B(\hat{\mathbf{n}})$.

Secondly, the states of the two individual particles are *entangled*, which amounts to saying that the state $|\psi\rangle$ cannot be written as a product of two individual states. This can be seen from the general case of a ket $|\omega\rangle = n_1|a\rangle|b\rangle + n_2|a'\rangle|b'\rangle$, for which it is impossible to find two kets $|i\rangle$ and $|j\rangle$, each defined in the configuration spaces of the individual particles, such that $|\omega\rangle = |i\rangle \otimes |j\rangle$. The impossibility is easily revealed by writing out $|i\rangle = i_1|a\rangle + i_2|a'\rangle$ and $|j\rangle = j_1|b\rangle + j_2|b'\rangle$. Their direct product is:

$$|i\rangle|j\rangle = i_1 j_1|a\rangle|b\rangle + i_1 j_2|a\rangle|b'\rangle + i_2 j_1|a'\rangle|b\rangle + i_2 j_2|a'\rangle|b'\rangle \tag{7}$$

From the original expression for $|\omega\rangle$ it can be deduced that $n_1 = i_1 j_1 \neq 0$ and $n_2 = i_2 j_2 \neq 0$, but also that $i_1 j_2 = i_2 j_1 = 0$. This latter fact implies that at least one of the coefficients i_1 and j_2 is zero, and one of i_2 and j_1. But then not both n_1, n_2 can be nonzero.

Since the system is invariant under rotation the kets $|\pm\rangle$ can be taken to be along any arbitrary direction $\hat{\mathbf{a}}$:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\hat{\mathbf{a}}, +\rangle|\hat{\mathbf{a}}, -\rangle - |\hat{\mathbf{a}}, -\rangle|\hat{\mathbf{a}}, +\rangle) \tag{8}$$

If a direction $\hat{\mathbf{b}}$ is defined to be along the polar axis, θ being the polar angle of $\hat{\mathbf{a}}$ and ϕ the azimuthal angle, then

$$|\hat{\mathbf{a}},+\rangle = \cos\frac{\theta}{2}e^{-i\frac{\phi}{2}}|+\rangle + \sin\frac{\theta}{2}e^{i\frac{\phi}{2}}|-\rangle \tag{9}$$

$$|\hat{\mathbf{a}},-\rangle = -\sin\frac{\theta}{2}e^{-i\frac{\phi}{2}}|+\rangle + \cos\frac{\theta}{2}e^{i\frac{\phi}{2}}|-\rangle \tag{10}$$

The invariance under rotation is apparent from the substitution of these equations into

$$|\psi_{\hat{\mathbf{a}}}\rangle = \frac{1}{\sqrt{2}}(|\hat{\mathbf{a}},+\rangle|\hat{\mathbf{a}},-\rangle - |\hat{\mathbf{a}},-\rangle|\hat{\mathbf{a}},+\rangle) \tag{11}$$

$$= \frac{1}{\sqrt{2}}(|+\rangle|-\rangle - |-\rangle|+\rangle) = |\psi\rangle \tag{12}$$

If $\hat{\mathbf{b}}$ is along the polar axis, then the coordinates can be chosen such that the azimuthal angle ϕ vanishes. The expression for the kets $|\hat{\mathbf{a}},\pm\rangle$ is then

$$|\hat{\mathbf{a}},+\rangle = \cos\frac{\theta}{2}|\hat{\mathbf{b}},+\rangle + \sin\frac{\theta}{2}|\hat{\mathbf{b}},-\rangle \tag{13}$$

$$|\hat{\mathbf{a}},-\rangle = -\sin\frac{\theta}{2}|\hat{\mathbf{b}},+\rangle + cos\frac{\theta}{2}|\hat{\mathbf{b}},-\rangle \tag{14}$$

Substitution into 11 yields:

$$\begin{aligned}|\psi\rangle = \frac{1}{\sqrt{2}}(&-\sin\frac{\theta}{2}|\hat{\mathbf{a}},+\rangle|\hat{\mathbf{b}},+\rangle + \cos\frac{\theta}{2}|\hat{\mathbf{a}},+\rangle|\hat{\mathbf{b}},-\rangle \\ &- \cos\frac{\theta}{2}|\hat{\mathbf{a}},-\rangle|\hat{\mathbf{b}},+\rangle - \sin\frac{\theta}{2}|\hat{\mathbf{a}},-\rangle|\hat{\mathbf{b}},-\rangle)\end{aligned} \tag{15}$$

And thus the probabilities are obtained for each of the combinations of outcomes for the individual spin measurements:

$$P_{\psi,\hat{\mathbf{a}}\hat{\mathbf{b}}}(++) = P_{\psi,\hat{\mathbf{a}}\hat{\mathbf{b}}}(--) = \frac{1}{2}\sin^2\frac{\theta}{2} \tag{16}$$

$$P_{\psi,\hat{\mathbf{a}}\hat{\mathbf{b}}}(+-) = P_{\psi,\hat{\mathbf{a}}\hat{\mathbf{b}}}(-+) = \frac{1}{2}\cos^2\frac{\theta}{2} \tag{17}$$

If the product of the spin measurements is considered, the following holds for its expectation value, for $\pm\pm = \{++,--\}$ and $\pm\mp = \{+-,-+\}$ (a notation that will be used throughout this paper):

$$\begin{aligned} E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) &= P_{\psi,\hat{\mathbf{a}}\hat{\mathbf{b}}}(\pm\pm) - P_{\psi,\hat{\mathbf{a}}\hat{\mathbf{b}}}(\pm\mp) = \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2} \\ &= -\cos\theta = -\hat{\mathbf{a}}\cdot\hat{\mathbf{b}} \end{aligned} \tag{18}$$

In particular, the perfect anti–correlation is found in the special case $E(\hat{\mathbf{a}}, \hat{\mathbf{a}}) = -1$.

3.3 EPR Incompleteness Argument

Einstein, Podolsky and Rosen presented the experiment that was described above in conjunction with their argument for the incompleteness of quantum mechanics. The argument proceeded from a number of assumptions that they made explicit. Below, they will be applied to the experiment immediately to yield the conclusion that they felt was inevitable.

Firstly, due to *perfect correlation*, if the spin of one particle is measured along an arbitrary direction, then the spin of the other along the same direction is known to be of equal magnitude but opposite sign. Secondly, due to *locality*, the measurement of the one particle could not have disturbed the other, which is space–like separated from it. The assumption of *realism* consequently states that any physical quantity that can be predicted with certainty without disturbing the system must have a counterpart in physical reality. In the particular example the reasoning is as follows: since the outcome of the measurement of the other particle is known (due to the fact that the one particle was measured along the same, arbitrary, direction and is certain to be opposite) there must be an element of physical reality corresponding to it. In other words, there must be something out there in the physical world to which this prediction corresponds. Due to locality, we are committed to recognise that this element of physical reality must be in the particle and that it was there all along, since measurement of the other particle could not have disturbed it.

Finally, the assumption of *completeness* entails that all that which is counted towards physical reality should be represented in a physical theory, in order for the latter to be considered complete. On the basis of this assumption, EPR felt quantum mechanics should be judged incomplete, since the argument seems to have persuaded that these particles possessed the property of having a certain spin along all possible directions (since after an arbitrary time the spin of particle one could be measured along any direction, and then the spin of the other particle would be known, and thus real and already present).

3.4 Bell's Inequalities

However, Bell derived in 1964 a set of inequalities that showed that the assumptions that EPR thought beyond any reasonable doubt are, in the light of the quantum mechanical predictions, not sound, and therewith exposed one of the most intriguing sides of quantum mechanics.

Bell's argument was that the four assumptions expressed by EPR are equivalent to the postulation of the existence of a completely state, completely specified, and denoted by a unit of information λ, where $\lambda \in \Lambda$, and Λ representing the set of all thinkable states the system can be in. For instance, equation 6 would, by the virtue of the assumptions, represent a abstract — and statistically specified — superposition of real states, each in itself completely determined (by assumption each particle has a well defined spin along any direction, either up or down, and even though it is not known which of the two is the case, the other particle must have an equally well defined opposite spin along the same direction — the degree of freedom of this system is the spin of one of the two particles along any direction). Λ then represents the set of all these possible completely determined states and λ just one particular one.

For example, for the two particles in the singlet state, taking only one direction into account, a particular λ_p is contains all information about the, assumed non–superimposed, spin value of each particle. In one case, λ_p specifies that in this situation the spin of particle 1 is up along the direction under consideration, and the spin of particle 2 is down along the same direction. Conversely, λ_q specifies the case, in which particle 1 has spin down along one direction and particle 2 thus spin up. The general $\Lambda = \{\lambda_p, \lambda_q\}$ contains all possible λ. Crucially, (1) this model does not allow the possibility of any of the particles not having either spin up or down, i.e. being in a superposition of states, however (2) in its generality it is the only model conceivable in which a system is "completely specified."

Consequently, the outcome of the spin measurements along any of the direction, as formulated in the form of the functions $A_\lambda(\hat{\mathbf{a}})$ and $B_\lambda(\hat{\mathbf{b}})$ (each depending only on their corresponding direction of measurement $\hat{\mathbf{a}}$ or $\hat{\mathbf{b}}$ due to locality and the assumption that these directions can be chosen and changed at any time) would be completely specified by λ only, since this a reformulation of what is understood by a system being "completely specified."

The expectation value of the product of the spin measurements is then given by an integral over all possible states (since in the quantum mechanical system there is not one particular completely specified state which accounts for the configuration of the state vector as it was given), with their absolute probability denoted by $\rho(\lambda)$.

$$E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = \int_\Lambda A_\lambda(\hat{\mathbf{a}}) B_\lambda(\hat{\mathbf{b}}) d\rho \tag{19}$$

The perfect correlation poses an important constraint to this expression, and it is that $E(\hat{\mathbf{a}}, \hat{\mathbf{a}}) = -1$. Since $A_\lambda(\hat{\mathbf{a}}), B_\lambda(\hat{\mathbf{b}}) = \pm 1$, it must then be that all $A_\lambda(\hat{\mathbf{a}}) = -B_\lambda(\hat{\mathbf{a}})$. Thus equation 19 can be rewritten as

$$E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = -\int_\Lambda A_\lambda(\hat{\mathbf{a}}) A_\lambda(\hat{\mathbf{b}}) d\rho \tag{20}$$

Then, since for all $\hat{\mathbf{n}}$, $A_\lambda(\hat{\mathbf{n}}) = \pm 1$, $A_\lambda(\hat{\mathbf{n}})$ equals its inverse, the following holds:

$$\begin{aligned} E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) - E(\hat{\mathbf{a}}, \hat{\mathbf{c}}) &= \int_\Lambda [-A_\lambda(\hat{\mathbf{a}}) A_\lambda(\hat{\mathbf{b}}) + A_\lambda(\hat{\mathbf{a}}) A_\lambda(\hat{\mathbf{c}})] d\rho \\ &= \int_\Lambda [-A_\lambda(\hat{\mathbf{a}}) A_\lambda(\hat{\mathbf{b}})][1 + \frac{A_\lambda(\hat{\mathbf{a}}) A_\lambda(\hat{\mathbf{c}})}{-A_\lambda(\hat{\mathbf{a}}) A_\lambda(\hat{\mathbf{b}})}] d\rho \\ &= \int_\Lambda [-A_\lambda(\hat{\mathbf{a}}) A_\lambda(\hat{\mathbf{b}})][1 - A_\lambda(\hat{\mathbf{b}}) A_\lambda(\hat{\mathbf{c}})] d\rho \end{aligned} \tag{21}$$

Since for all $\hat{\mathbf{n}}, \hat{\mathbf{m}}$, $A_\lambda(\hat{\mathbf{n}}) = \pm 1$, it follows that $|A_\lambda(\hat{\mathbf{n}}) A_\lambda(\hat{\mathbf{m}})| = 1$ and also that $[1 - A_\lambda(\hat{\mathbf{n}}) A_\lambda(\hat{\mathbf{m}})] \geq 0$ and thus $|1 - A_\lambda(\hat{\mathbf{n}}) A_\lambda(\hat{\mathbf{m}})| = 1 - A_\lambda(\hat{\mathbf{n}}) A_\lambda(\hat{\mathbf{m}})$, taking the absolute value of both sides of 21 yields:

$$\begin{aligned} |E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) - E(\hat{\mathbf{a}}, \hat{\mathbf{c}})| &\leq \int_\Lambda [1 - A_\lambda(\hat{\mathbf{b}}, \hat{\mathbf{c}})] d\rho = \int_\Lambda d\rho + E(\hat{\mathbf{b}}, \hat{\mathbf{c}}) \\ &\leq 1 + E(\hat{\mathbf{b}}, \hat{\mathbf{c}}) \end{aligned} \tag{22}$$

This inequality can be shown to be violated by particular choices of the measured $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$, using the derived expression for the expectation value $E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = -\cos\theta$. For $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$ all being in the same plane, with angles $\frac{\pi}{3}$ between $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ on the one hand, and $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ on the other, we have that

$$E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = E(\hat{\mathbf{b}}, \hat{\mathbf{c}}) = -\cos\frac{\pi}{3} = -\frac{1}{2} \qquad E(\hat{\mathbf{a}}, \hat{\mathbf{c}}) = -\cos\frac{2\pi}{3} = +\frac{1}{2} \tag{23}$$

Then the inequality expressed in 22 is violated:

$$|E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) - E(\hat{\mathbf{a}}, \hat{\mathbf{c}})| = 1 > \frac{1}{2} = 1 + E(\hat{\mathbf{b}}, \hat{\mathbf{c}}) \tag{24}$$

3.5 Review of the Argumentation

Schematically the Einstein, Podolsky and Rosen argument can be represented as follows:

1. **Assumption** Perfect Correlation
2. **Assumption** Locality
3. **Assumption** Realism
4. **Conclusion** A particle in the EPR experiment has well defined spin along all directions
5. **Assumption** Completeness
6. **Observation** There exist no quantum mechanical state a particle can be in, such that it simultaneously has well–defined spin along all directions.
7. **Conclusion** Quantum mechanics is not a complete theory.

Similarly, from Bell's Theorem the following argument can be deduced:

1. **Observation** The EPR Assumptions imply the existence of the functions $A_\lambda(\hat{\mathbf{a}}), B_\lambda(\hat{\mathbf{b}}) = \pm 1$.
2. **Deduction** From the existence of these functions, the inequality 22 can be derived.
3. **Observation** The quantum mechanical prediction violates this inequality.
4. **Conclusion** Either quantum mechanics is wrong in its predictions, or the EPR assumptions will have to be abandoned or at least reformulated.

As to the latter point, ultimately experimental verification could be called in to test that the quantum mechanical prediction is indeed correct.

4 Mermin's Picture of the EPR Experiment

4.1 Experimental Setup and Results

David Mermin describes in his article "Is the moon there when nobody looks? Reality and the quantum theory"[4] and his book "Boojums all the way through: communicating science in a prosaic age"[5] the EPR experiment in more abstract terms, in order for it to be (1) accessible to non–physicists and (2) instructive in the sense that it shows the fundamental working of the argument based on Bell's Theorem without abstract mathematics.

Mermin's picture of the EPR experiment consists of a source and two detectors, placed on opposite sides of the source. The source emits, at any time the experimenter requires it so, two entities that consequently arrive at each of the two detectors. The detectors earn the name by their reaction to the arriving entity: each has a red and a green light, and upon the arrival of the entity either one of the two flashes, but never both. Thus, by temporarily placing an obstruction between the source and one of the detectors it can be seen from the fact that now neither of the lights flashes that there was indeed an interaction. That the entity is finite in space can be seen from obstructing the path temporarily at particular places and determining whether the detectors react or not. Additionally, the two detectors each have a switch that can be set in three possible ways, from now on referred to as way 1, 2 and 3. At this point it is important to emphasise that there are no connections between the detectors, nor information exchange. Thus the only thing that connects them is that the two entities have the source as their common origin.

A run of the experiment is called the act of persuading the source to emit its two entities and observing which of the lights flash consequently.

It is very important at this stage not to think in terms of spin measurements, particles and Stern–Gerlach devices, but rather in the abstract terminology used so far, such as source, entities and detectors, for the strength of Mermin's picture is that it renders Bell's Theorem clearer by casting off all essentially irrelevant physical details.

In addition to this specification of the experimental configuration, the argument requires observation of patterns in the experimental data derived from executing the experiment a sufficiently large number of times. Mermin notes that the relevant observations that can be made about the proceeding of the experiment are the following — and at this point they will not be physically explained, as they need not be, but merely pointed out and assumed to reflect the physical reality of the hypothetical experiment —:

1. If the switch settings are the same, the lights always flash the same colour.

2. Overall (regardless of the switch settings) the flashing pattern is completely random.

In particular, this latter observation implies that the chance that the same colours flash is equal to the chance that different colours flash: $P(RR) = P(RG) = P(GR) = P(GG)$.

4.2 Representation of Instruction Sets

Mermin's question is then whether the observations 1 and 2 can be explained by the existence of *instruction sets*. Instruction sets are the abstract representation of the functions $A_\lambda(\hat{\mathbf{a}}), B_\lambda(\hat{\mathbf{b}}) = \pm 1$ — and they play the same role in the argument of Mermin as they did in the argument on the basis of Bell's Theorem —; they are understood to (1) be contained in each of the carriers (though not necessarily are the same in both carriers) and to (2) represent the complete determination of which light should flash on the detector as a result of the corresponding particle's arrival. Mermin's argument is that these instruction sets are impossible given the observations, and thus that the system is not completely specified.

Mermin's picture of an instruction set for a particle will in this paper be represented by a three-component vector of which values of the components represent the colours the light will flash if the associated switch is set on the detector it arrives at. The convention in this paper will be to represent a red light by -1 and a green light by $+1$. There are then $2^3 = 8$ possible instruction sets, and they are:

$$I \in \{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \} \tag{25}$$

Each switch setting is represented by the row vector, which has zeroes in all rows except in one and is thus of unit norm. Which colour will flash when a particle carrying a particular instruction set arrives at a detector is given by the dot product of the instruction set vector with the unit vector corresponding to the switch settings of the particular detector. For instance:

$$S_1 \cdot I_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = -1 \tag{26}$$

4.3 Impossibility of Instruction Sets

Finally, Mermin's argument is that the observations 1 and 2 cannot *both* be explained by assuming that the entities carry instruction sets.

Due to observation 1 it is apparent that the two entities, if at all equipped with an instruction set, must certainly be carrying the same one. This observation, since it is stated universally and not statistically, is a good starting point to elaborate the formalism introduced so far. The product of the ± 1 associated to the light flashing on each of the detectors is

$$R = \mathbf{S} \odot \mathbf{I} = (S_1 \cdot I_1)(S_2 \cdot I_2) \tag{27}$$

Here I have defined the operator $\odot$ as a vector operation such that:

$$\mathbf{S} \odot \mathbf{I} \equiv \prod_i S_i \cdot I_i \tag{28}$$

Observation 1 then implies that for $S_1 = S_2$ we have that $R = (\pm 1)^2 = +1$, and this can only be for all $\mathbf{I}$ and $\mathbf{S}$ if $I_1 = I_2$. In other words, the particles must be carrying the same instruction sets.

Next, the possible instruction sets introduced in equation 25, can be divided into two sorts: (1) the [3 : 0]–sets, which contain only the 2 instruction sets for which the flashing pattern is independent of the switch setting, and (2) the [2 : 1]–sets, which contain the other 6 instruction sets, which have the shared property that in two of the tree cases the light will flash a certain colour, and in the other case it will flash the other colour.

Now in the [3 : 0]–case, it is certain that both lights will flash the same colour. In the [2 : 1]–case, lights will in $\frac{5}{9}$ cases flash the same colour. This is easily verified: in total there are $3 \cdot 3 = 9$ possible configurations of the two detectors together, and in 3 of these, they have the same setting and thus, regardless of which instruction set both particles carry, lights will flash the same colour. Furthermore, it was established before that each of the instruction sets of the [2 : 1]–sort must have one (and of course only one) of the colours appearing twice. Thus there are, in addition to the 3 cases in which the switch settings are the same, always 2 cases in which the switch settings are not equal, but each one is on one of the settings corresponding to an appearance of the double colour in the instruction set. For example, the [2 : 1] instruction set from equation 28 will yield the same lights if the switch settings are $\{11, 22, 33, 23, 32\}$ which are 5 out of 9 cases.

Consequently, even an arbitrary combination these instruction sets will always yield for at least $\frac{5}{9}$ of the cases the same lights flashing (with it being equal to $\frac{5}{9}$ for inclusion of only [2 : 1]–cases) , which is contrary to observation 2. Thus it is impossible to represent the system by instruction sets conforming to both observations 1 and 2. This leads Mermin to conclude, in analogy with the argument from Bell's Theorem, that the flashing pattern of the lights cannot be caused by instruction sets. If it would be assumed that (1) these mysterious detectors can indeed be constructed to physically function as has been described, (2) that the entity travelling from the source to the detectors is indeed a particle, and (3) that they measure spin along any three directions specified by the switch settings, then it seems valid to formulate conclusions about the physical reality as it appears to scientific inquiry.

However, this will not be done here, since Mermin proposed a very similar picture for a more recent gedankenexperiment designed by Greenberger, Horne and Zeilinger, which will be presented in section 5 and discussed in more detail.

5 Mermin's Picture of the GHZ Experiment

5.1 Experimental Setup and Results

Like in Mermin's picture of the EPR experiment, the GHZ experiment[6] consists of a source that can be made to emit entities, but this time not two, but three, each heading for its own detector. The detectors, in turn, are simplified in that they have only two instead of three switch settings, denoted by 1 and 2, and they still upon arrival of the carrier flash their red or green light, but not both. Again, it is very essential that there are no connections between the detectors, so that the only thing that they share is propagated by the entities that travel from the (common) source.

When performing this experiment for switches set randomly the following to observational facts can be established:

1. If all detectors are set to 1, then an even number of red lights will flash.
2. If one of the detectors is set to 1 and the other two to 2, then an odd number of red lights will flash.

Since there are in total 3 lights, an odd number of lights flashing one colour implies that an even number of lights flash the other colour:

$$\mathrm{odd}[R] = \mathrm{even}[G] = \neg\mathrm{even}[R] \tag{29}$$

5.2 Representation of Instruction Sets

Again, the red and green lights are represented by ± 1, respectively. An instruction set means a complete specification of the flashing pattern that will result from and is assumed to be completely specified by the release of the entities from the source. Each particle again will be assumed to carry its own instruction set and the collection of the three instruction set of each of the particles in a particular run will be referred to as the instruction set corresponding to that particular run (this time, there is no reason to assume that they carry the same instruction set, and thus the possibility will remain open) It will be represented as a 2×3–matrix, where the columns correspond to the different detectors (and thus the different particles) and the rows correspond to their possible switch settings. An example is:

$$\mathbf{I} = \begin{bmatrix} I_1 & I_2 & I_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \tag{30}$$

Each I_j can take 4 different configurations, thus there are in total 4^3 possible instruction sets.

A switch setting is denoted by a vector with two entries, one of which is 0 and the other 1. The result of a run, denoted by R, is then represented as the product of the ± 1 associated with each of the separate detectors:

$$R = \mathbf{S} \odot \mathbf{I} = (S_1 \cdot I_1)(S_2 \cdot I_2)(S_3 \cdot I_3) \tag{31}$$

An odd number of red lights flashing thus implies $R = -1$ whereas an even number implies $R = +1$.

5.3 Impossibility of Instruction Sets

Mermin's Argument is then, analogous with his discussion of the EPR experiment, that there can be no instruction sets of the form described above, that can account for the observations 1 and 2. Again, the starting point is the more restrictive of the two observations, which in this case is observation 2.

In the formal notation introduced before, the runs that fall under observation 2 are characterised by switch setting configurations of the form $[122] = \{122, 212, 221\}$. Now, considering these cases one by one, the restrictions that each of these pose on the instruction sets will be analysed, and thus it is expected that for each of them a number of the originally 4^3 possible instruction sets will have to be eliminated, since not all of the possible instruction sets yield an odd number of red light flashes for a particular switch setting. An odd number of red flashes practically means that either 1 or 3 red lights flash; in the former case there being 3 possible flash patterns (in the notation used before, these are the results $[-1, 1, 1]$, and in the latter case there being only 1 way.

Firstly the switch configuration 122 will be considered. The requirement that in this case an odd number of red lights must flash means that the instruction sets will have to be of either of the following form (where empty entries denote entries which can still take any value, and therefore these matrices each represent in fact 2^3 matrices, since they have 3 empty entries):

$$I_{122} = \{\begin{bmatrix} -1 & & \\ & -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & & \\ & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & & \\ & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & & \\ & 1 & -1 \end{bmatrix}\} \tag{32}$$

Secondly, the switch configuration 212 will be considered. Possible instruction sets are then:

$$I_{212} = \{\begin{bmatrix} & -1 & \\ -1 & & -1 \end{bmatrix}, \begin{bmatrix} & 1 & \\ -1 & & 1 \end{bmatrix}, \begin{bmatrix} & 1 & \\ 1 & & -1 \end{bmatrix}, \begin{bmatrix} & -1 & \\ 1 & & 1 \end{bmatrix}\} \tag{33}$$

Likewise, for 221 the possible instruction sets are:

$$I_{221} = \{\begin{bmatrix} & & -1 \\ -1 & -1 & \end{bmatrix}, \begin{bmatrix} & & 1 \\ -1 & 1 & \end{bmatrix}, \begin{bmatrix} & & 1 \\ 1 & -1 & \end{bmatrix}, \begin{bmatrix} & & -1 \\ 1 & 1 & \end{bmatrix}\} \tag{34}$$

Finally, the instruction sets that conform to the observation 2 are given by the intersection of these sets, and explicitly it is given by:

$$\mathbf{I_2} \in I_{112} \cap I_{121} \cap I_{211}$$
$$= \{ \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix},$$
$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \}$$

Also, the collection of instruction sets that satisfy all instances of observation 1 are given by

$$\mathbf{I_1} \in I_{112} \cap I_{121} \cap I_{211}$$
$$= \{ \begin{bmatrix} -1 & -1 & 1 \\ & & \end{bmatrix}, \begin{bmatrix} -1 & 1 & -1 \\ & & \end{bmatrix}, \begin{bmatrix} 1 & -1 & -1 \\ & & \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ & & \end{bmatrix} \}$$

It is not difficult to see that they do not intersect — in other words, there are no instruction sets possible that satisfy both the constraint of having to conform to observation 1 and observation 2 — since each of the instruction sets in I_1 has an even number of -1–values in the first row (by definition) and each one in I_2 has an odd number of -1–entries (as a result of the derivation), which makes them mutually exclusive.

$$\mathbf{I_1} \cap \mathbf{I_2} = \emptyset \tag{35}$$

5.4 Overview of Argumentation

The arguments presented so far have been the following:

1. **Assumption** If the entity has definite properties which the detector measures — and if the property it measures is determined by the switch setting — then these properties can be represented conceptually by instruction sets.

2. **Derivation** The existence of instruction sets is incompatible with the observed phenomena.

3. **Conclusion** Therefore there cannot exist instruction sets.

4. **Conclusion** Therefore the entity does not have definite properties.

In this paper another, more physical argument to the same end will be presented, on the basis of a more detailed description of the mysterious boxes that Mermin introduced.

1. **Assumption** If the particle has definite properties which the detector measures — and if the property it measures is determined by the switch setting — then these properties can be represented conceptually by instruction sets.

2. **Assumption** Instruction sets can in turn be represented by a series of functions $A_\lambda(\hat{\mathbf{a}}), B_\lambda(\hat{\mathbf{b}}), C_\lambda(\hat{\mathbf{c}})$ dependent on λ in their full specification but then for obtaining the concrete (physical) ± 1 value only on the corresponding local variables.

3. **Derivation** These functions $A_\lambda(\hat{\mathbf{a}}), B_\lambda(\hat{\mathbf{b}}), C_\lambda(\hat{\mathbf{c}})$ cannot exist in conjunction with the phenomena predicted by quantum mechanics and experimentally observed.

4. **Conclusion** Therefore there cannot exist instruction sets.

5. **Conclusion** Therefore the particle does not have definite properties.

In fact, the same argument could hold without the intervention of the instruction sets, but it has been chosen to include these, to make clear how this argument relates to Mermin's more abstract argument.

6 The Physics of the GHZ Experiment

6.1 Experimental Setup

A particle will be prepared such that it decays into three particles of equal mass and energy. If the initial particle has mean momentum zero, then momentum– and energy conservation dictate that the three particles it decays into must be emitted 120° apart. The initial particle is what in Mermin's picture was the source, and it is surrounded by a demarcation with only six openings, which can be divided into two sets of three openings, each 120° apart. The openings in one of these two sets will be called $\{a, b, c\}$ and those in the other $\{a', b', c'\}$. In the configuration thus presented the particles go through one of these two sets of openings and not through a combination of

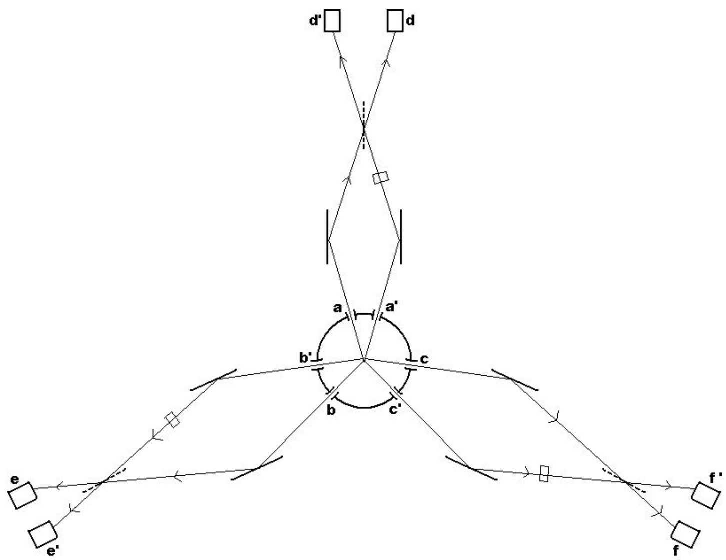

Figure 1: Configuration of GHZ–experiment

them. Next, the configuration for each particle is completely the same, and it will be referred to as particle i or particle i' depending on whether it went through the nonprimed or primed opening, respectively.

The latter particle is first subjected to a phase plate which shifts its phase by a value that can be specified by the experimenter and is denoted by ϕ_i (A phase shift of this kind could for instance be achieved by subjecting the particle to a lower potential — for instance lowering it in the gravitational field — which would give it a larger wavelength and therefore cause a phase shift when brought back to the original potential). Both beams i and i' encounter a mirror so that they are made to intersect again at a 50-50–beam splitter, which, for each particle, is equally likely to reflect it or to leave it through. A particle that went through i, (1) if not reflected at the beam splitter, would end up at an associated detector $p \in \{d, e, f\}$ (which is also where i' would end up if it *is* reflected), or (2) if reflected at the beam splitter, would end up at an associated detector p' (which is also where i' ends up if it is *not* reflected).

6.2 Quantum Physical Model

The state vector is given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|a\rangle_1|b\rangle_2|c\rangle_3 + |a'\rangle_1|b'\rangle_2|c'\rangle_3). \tag{36}$$

Again, it is clearly entangled, since it cannot be written as a product of states of the individual particles in their corresponding state space.

For each of these particles, for instance a, the evolution is, according to the description given before:

$$|a\rangle_1 \rightarrow \frac{1}{\sqrt{2}}(|d\rangle_1 + i|d'\rangle_1) \tag{37}$$

$$|a'\rangle_1 \rightarrow \frac{1}{\sqrt{2}}e^{i\phi_1}(|d'\rangle_1 + i|d\rangle_1) \tag{38}$$

The imaginary unit in the coefficient is due to reflection. Its action is to rotate the wave function in the complex plane, just like a travelling wave on a rope is inversed (i.e. rotated, but then over an angle π) when reflected on a wall. The angle $\frac{\pi}{2}$ yields the rotation coefficient $e^{i\frac{\pi}{2}} = i$.

Thus the state vector of equation 36 can be expanded into:

$$\begin{aligned}
|\Psi\rangle = \quad & \tfrac{1}{4}[&& (|d\rangle_1|e\rangle_2|f\rangle_3 + i|d\rangle_1|e\rangle_2|f'\rangle_3 \\
& && +i|d\rangle_1|e'\rangle_2|f\rangle_3 - |d\rangle_1|e'\rangle_2|f'\rangle_3 \\
& && +i|d'\rangle_1|e\rangle_2|f\rangle_3 - |d'\rangle_1|e\rangle_2|f'\rangle_3 \\
& && -|d'\rangle_1|e'\rangle_2|f\rangle_3 - i|d'\rangle_1|e'\rangle_2|f'\rangle_3) + \\
& e^{i\sum_j \phi_j}(&& -i|d\rangle_1|e\rangle_2|f\rangle_3 - |d\rangle_1|e\rangle_2|f'\rangle_3 \\
& && -|d\rangle_1|e'\rangle_2|f\rangle_3 + i|d\rangle_1|e'\rangle_2|f'\rangle_3 \\
& && -|d'\rangle_1|e\rangle_2|f\rangle_3 + i|d'\rangle_1|e\rangle_2|f'\rangle_3 \\
& && +i|d'\rangle_1|e'\rangle_2|f\rangle_3 + |d'\rangle_1|e'\rangle_2|f'\rangle_3)]
\end{aligned} \tag{39}$$

This expansion of the state vector allows computation of the probability that certain particles end up at certain detectors. More precisely, for each pair of a primed and nonprimed detector the particle can eventually end up in the primed or the nonprimed detector, and therefore there are 2^3 possible results of a run.

The coefficients in equation 39 give the probability that all particles end up at the nonprimed detectors is found by taking the absolute value:

$$\begin{aligned} P_{def}(\phi_1,\phi_2,\phi_3) &= \left(\frac{1}{4}\right)^2 |1 - ie^{\sum_j \phi_j}|^2 \\ &= \frac{1}{16}|1 - i(\cos\chi + i\sin\chi)|^2 \\ &= \frac{1}{16}\left((1+\sin\chi)^2 + (-\cos\chi)^2\right) \\ &= \frac{1}{16}(2 + 2\sin\chi) \end{aligned} \tag{40}$$

Likewise, for the same case, except for the a, a'–particle, which ends up at the primed detector d';

$$\begin{aligned} P_{d'ef}(\phi_1,\phi_2,\phi_3) &= \left(\frac{1}{4}\right)^2 |i - e^{\sum_j \phi_j}|^2 \\ &= \frac{1}{16}|i - (\cos\chi + i\sin\chi)|^2 \\ &= \frac{1}{16}\left((1-\sin\chi)^2 + (-\cos\chi)^2\right) \\ &= \frac{1}{16}(2 - 2\sin\chi) \end{aligned} \tag{41}$$

The effectively, the difference between these two is indeed in the sign of the cross term, since for the a–beam, now the reflected part is taken, and for the a'–beam the unreflected part is taken, and thus the imaginary unit i is exchanged.

There then exist four of these coupled probabilities and they are

$$P_{def} - P_{d'ef} = P_{de'f'} - P_{de'f} = P_{d'ef'} - P_{d'ef} = P_{d'e'f} - P_{d'e'f'} = \frac{1}{4}\sin(\phi_1 + \phi_2 + \phi_3) \tag{42}$$

This greatly facilitates the computation of the expectation value. For its computation, a value -1 is taken to represent each particle that ended up at its primed detector, and a value $+1$ is taken for each particle that ended up at its nonprimed detector. The result of the run is then expressed as the product of these outcomes. Noteworthy is then, that for an even number of particles ending up at their primed detectors, the result $R = +1$ and for an odd number the result is $R = -1$.

The expectation value is then

$$\begin{aligned} E(\phi_1, \phi_2, \phi_3) &= P_{def} - P_{def'} - P_{de'f} + P_{de'f'} \\ &\quad -P_{d'ef} + P_{d'ef'} + P_{d'e'f} - P_{d'e'f'} \\ &= \sin(\phi_1 + \phi_2 + \phi_3) \end{aligned} \tag{43}$$

The perfect correlations are then obtained in the cases where the expectation value $E(\phi_1, \phi_2, \phi_3) = \pm 1$, since then the probabilities of precisely half of the possible cases vanish (for instance, if $E(\phi_1, \phi_2, \phi_3) = 1 = \sin(\phi_1+\phi_2+\phi_3)$ then, by equation 41, $P_{d'ef}$ vanishes). Furthermore, it is noteworthy that the expectation value is essentially not a function of three variables, but only one: $\sum_{j=1}^{3} \phi_j$.

Thus the model of quantum mechanics presented here on predicts the following:

$$\phi_1 + \phi_2 + \phi_3 = \frac{\pi}{2} \quad \rightarrow E(\phi_1, \phi_2, \phi_3) = -1 \tag{44}$$

$$\phi_1 + \phi_2 + \phi_3 = \frac{3\pi}{2} \quad \rightarrow E(\phi_1, \phi_2, \phi_3) = +1 \tag{45}$$

6.3 Connection with Mermin's Picture

This problem has earlier been introduced in terms of abstract black boxes and it would be at least satisfying if there could be provided a formal correspondence between the physical system introduced in this section, and Mermin's black boxes configuration. This task seems not impossible. The similarity in configuration of the two problems suggests that the following should be equated: (1) the source is the particle that decays, (2) each of the detectors in Mermin's picture corresponds to the two coupled primed and nonprimed detectors in the detailed system of this section — such that the primed detector is connected to one of the lights and the nonprimed to the other, (3) the entities carrying information to the detectors are particles that the initial particle has decayed into, (4) the switch implements the choice of phase shift in the phase plate that each primed beam passes through.

Firstly, since it has so far arbitrarily been chosen to represent a particle detected by a primed detector by -1 and a particle detected by a nonprimed detector by $+1$, a red flash by -1 and a green one by $+1$, it seems natural to connect the primed detector to the red light, such that this light will flash if the particle ends up in this detector and otherwise not. Secondly, the switches are thought to represent a choice between two different phase shifts, implemented by the phase plate, and represented for a particle i by ϕ_{i1} and ϕ_{i2}. There is then a total of four equations (or an disjunctive infinity of them, if the modulus is taken into account) with six unknowns arise from the experimental observations 1 and 2 required for Mermin's model (ϕ_{nm} being the phase shift corresponding to the "primed path" of particle n and m the corresponding switch setting):

$$111: \qquad \sum_{n\in\{a,b,c\}} \phi_{n1} = \frac{\pi}{2} \mod 2\pi \tag{46}$$

$$122: \qquad \sum_{n\in\{a,b,c\}} \phi_{n1} + (\phi_{b2} - \phi_{b1}) + (\phi_{c2} - \phi_{c1}) = \frac{3\pi}{2} \mod 2\pi \tag{47}$$

$$212: \qquad \sum_{n\in\{a,b,c\}} \phi_{n1} + (\phi_{a2} - \phi_{a1}) + (\phi_{c2} - \phi_{c1}) = \frac{3\pi}{2} \mod 2\pi \tag{48}$$

$$221: \qquad \sum_{n\in\{a,b,c\}} \phi_{n1} + (\phi_{a2} - \phi_{a1}) + (\phi_{b2} - \phi_{b1}) = \frac{3\pi}{2} \mod 2\pi \tag{49}$$

Solutions to this equation would imply the direct connection with Mermin's model. Due to the abundance of degrees of freedom, the angles corresponding to the switch set to 1 can be chosen freely, and in their choice only limited by equation 46. Additionally, the other equations specify that for any pair of two detectors, the sum of the difference, per detector, between the phase shifts corresponding to switch setting 1 and 2 should be an odd multiple of π.

An example of a set of phase shifts that satisfy these conditions (even without requiring the modulus over 2π):

$$\phi_{a1} = \frac{\pi}{2}, \quad \phi_{b1} = \phi_{c1} = 0, \quad \phi_{a2} = \pi, \quad \phi_{b2} = \phi_{c2} = \frac{\pi}{2} \tag{50}$$

6.4 Bell's Theorem

Even though this already provides a sufficient argumentation for the thesis that the particle does not have a fixed property (which property in this case could be referred to as its "primed"– or "nonprimed"–ness), another argument to the same end will be presented here.

It will be held here that equivalently to the postulation of instruction sets one can derive the necessary existence of a set of functions completely specifying the outcome of each of the measurements on the basis of λ and then only the local ϕ_i ($\{A_\lambda(\phi_1), B_\lambda(\phi_2), C_\lambda(\phi_3)\}$) that take values ± 1 from the assumptions that were made explicit by EPR. The quantum theoretical predictions then pose the following restrictions on these functions, which is hoped to be shown exhaustively in this section, cannot be satisfied:

$$\phi_1 + \phi_2 + \phi_3 = \frac{\pi}{2} \quad \rightarrow A_\lambda(\phi_1)B_\lambda(\phi_2)C_\lambda(\phi_3) = -1 \tag{51}$$

$$\phi_1 + \phi_2 + \phi_3 = \frac{3\pi}{2} \quad \rightarrow A_\lambda(\phi_1)B_\lambda(\phi_2)C_\lambda(\phi_3) = 1 \tag{52}$$

Taking two particular cases of the general equation 51 and using the fact that each of the functions equals its inverse:

$$\begin{aligned} A_\lambda(0)B_\lambda(\frac{\pi}{2})C_\lambda(0) &= -1 = A_\lambda(0)B_\lambda(0)C_\lambda(\frac{\pi}{2}) \\ B_\lambda(0)C_\lambda(0) &= B_\lambda(\frac{\pi}{2})C_\lambda(\frac{\pi}{2}) \end{aligned} \tag{53}$$

Substitution of 53 into the following particular case of equation 51:

$$A_\lambda(\frac{\pi}{2})B_\lambda(0)C_\lambda(0) = -1 \tag{54}$$

$$A_\lambda(\frac{\pi}{2})A_\lambda(\frac{\pi}{2})B_\lambda(\frac{\pi}{2}) = -1 \tag{55}$$

However, this is contrary to equation 52, which demanded

$$A_\lambda(\frac{\pi}{2})B_\lambda(\frac{\pi}{2})C_\lambda(\frac{\pi}{2}) = +1 \tag{56}$$

Thus a set of functions $\{A_\lambda(\phi_1), B_\lambda(\phi_2), C_\lambda(\phi_3)\}$ satisfying the conditions of both equations 51 and 52 cannot be found. A more physical conclusion would be that the system was not completely specified.

7 Conclusion

7.1 Properties, Instruction Sets, Functions

The discussion of this paper, after a general philosophical introduction, has proceeded from a most concrete, physical example, via Mermin's abstract gedankenexperiments, back to a detailed analysis of the physics of the GHZ–experiment.

In this paper several arguments have been presented that all lead up to the same conclusion. Interestingly, they seem to reveal a formal resemblance between properties, instruction sets and functions. As has been argued before, properties can be understood as being instruction sets, since, as was noted before, properties are typically defined in terms of the perceiver (whether human or device) and not first in terms of the objects they belong to. When considering the case of the EPR experiment, where the detectors allow the measurement of spin along all directions, the EPR assumptions lead up to the conclusion that there must exist for each particle and for each direction a property of the value of the spin along the infinity of possible directions. Likewise, instruction sets can be represented in the form of the functions $\{A_\lambda(\hat{\mathbf{a}}), B_\lambda(\hat{\mathbf{b}}), C_\lambda(\hat{\mathbf{c}})\}$ defined over precisely this domain of possible directions and taking the values ± 1.

Generally, on the basis of the discussion presented in this paper properties, instruction sets, and the set of functions $\{A_\lambda(\hat{\mathbf{a}}), B_\lambda(\hat{\mathbf{b}}), C_\lambda(\hat{\mathbf{c}})\}$ can be considered as conceptually equivalent.

7.2 Existence of Fixed Properties

The experiments presented in this paper have shown that there cannot exist a model incorporating the existence of a hidden variable λ that *a priori* can take different values $\lambda_n \in \Lambda$ but for each physical system is fixed, and completely determines the outcomes of all possible measurements.

Thus the only physically correct conclusion would be that on the level of particles, there does no longer exist such as thing as fixed properties. Mermin writes in his article "The (non)world (non)view of quantum mechanics"[7] that "[t]he correct attitude is that the concepts of position and velocity cease to have meaning at the atomic level."

Furthermore, since nevertheless fixed properties are observed (it is even predicted that it is impossible to directly observe a superposition of states) it must be that the act of measurement has itself persuaded the system to assume fixed properties.

7.3 Terms of Perception

Finally, on the basis of this discussion it seems correct to conclude that the postulation of the existence of a state vector containing the statistical information about the outcome of potential measurements, has, in the light of the adaptation of Berkeley's argument as presented in this paper, not quite transcended the thinking in terms of perception. For although a higher level of abstraction seems to have been reached — and in a certain sense has been — in the human understanding it is still essentially a statistical abstraction of a set of possible outcomes which are as always understood in terms of perception. The EPR experiment, and how natural its assumptions come, ultimately inconsistent or not, shows that the aim of understanding a state vector in itself, not in terms of perception, but as more than just an addition of states, has still not been achieved.

References

[1] G. Berkeley. *A Treatise Concerning the Principles of Human Knowledge.* ed. G.N. Wright, London, 1710,1843.

[2] D.M.Greenberger, M.A.Horne, A.Shimony, and A.Zeilinger. Bell's theorem without inequalities. *American Journal of Physics*, 58:1131–1143, december 1990.

[3] J. Locke. *An Essay Concerning Human Understanding.* Hackett Publishing Company, Inc., Indianapolis, 1690, 1999.

[4] N. D. Mermin. Is the moon there when nobody looks? reality and the quantum theory. *Physics Today*, pages 38–47, April 1985.

[5] N. D. Mermin. *Boojums all the way through: communicating science in a prosaic age.* Cambridge University Press, Cambridge, 1990.

[6] N. D. Mermin. Quantum mysteries revisited. *American Journal of Physics*, 58:731–734, August 1990.

[7] N. D. Mermin. The (non)world (non)view of quantum mehcanics. *New Literary History*, 23:855–875, 1992.

[8] R. Shankar. *Principles of quantum mechanics.* Kluwer Academic / Plenum Publishers, New York, 1994.