What makes a sentence be about the world? Towards a unified account of groundedness

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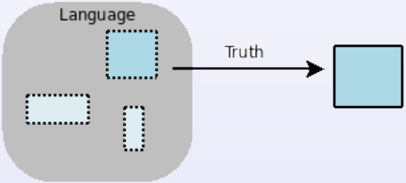

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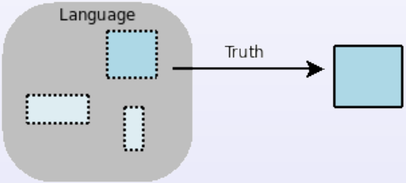

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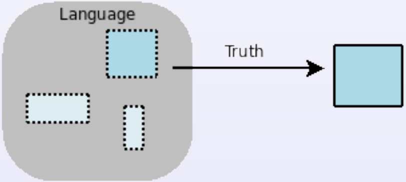

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Concept of "truth"

- Intuitively clear, yet
- Immediately problematic

Paradox and the concept of truth

Truth–predicate

A predicate Tr that applies to any (code of) sentence ϕ, such that

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Groundedness

Referring (in)directly to non–semantic states of affairs.

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- *Grounded sentences* are those that eventually obtain a truth value.

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- T–equivalences are required to hold only for grounded sentences.

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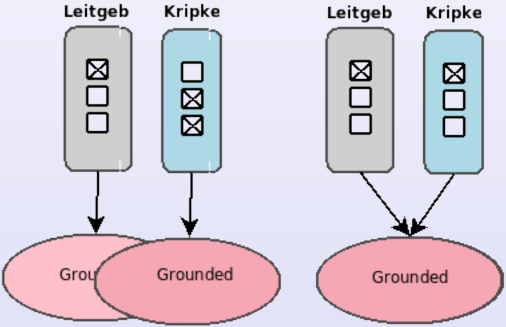
Hypothesis

There is one notion of groundedness, but Kripke and Leitgeb's *parameter settings* differ.



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- Cantini's $\boxed{\pm E'_\infty}$ includes Leitgeb's $\boxed{\Phi_{\mathsf{lf}}}$, but strictly.

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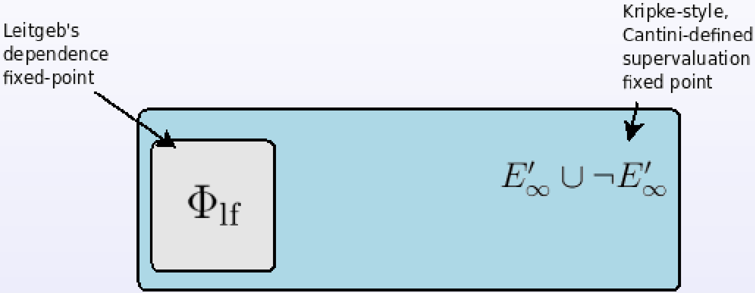

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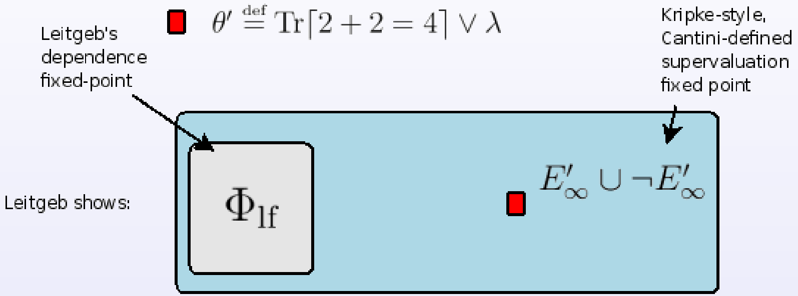

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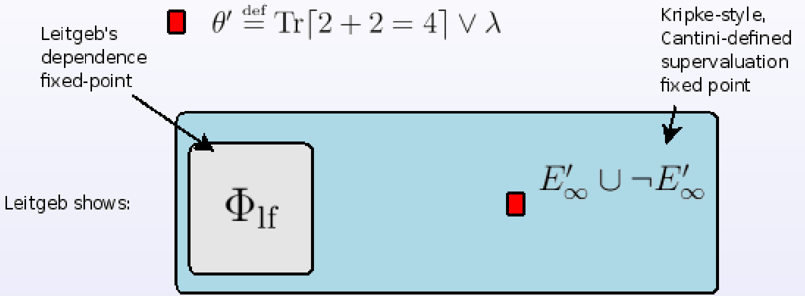

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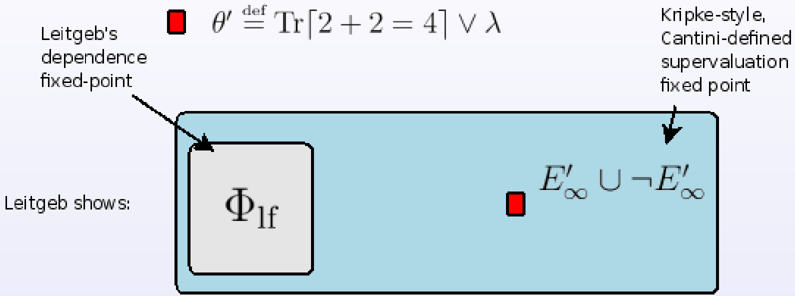

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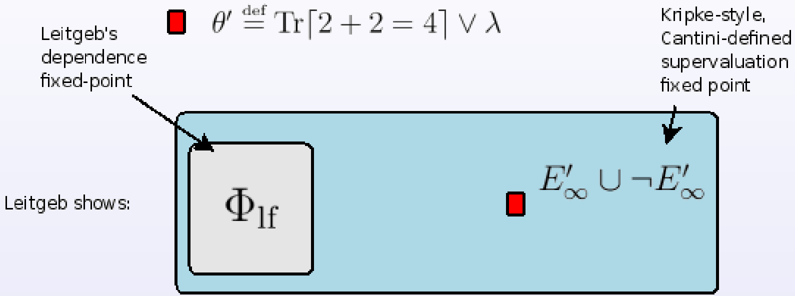

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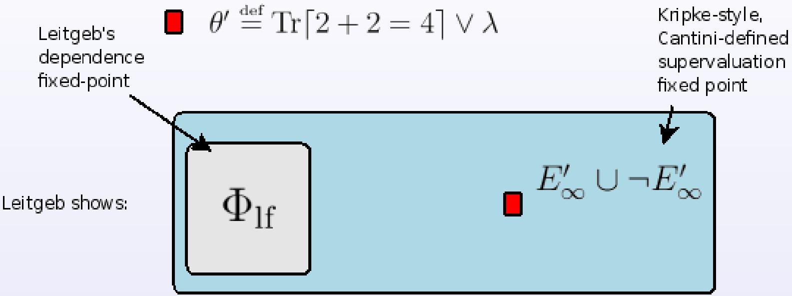

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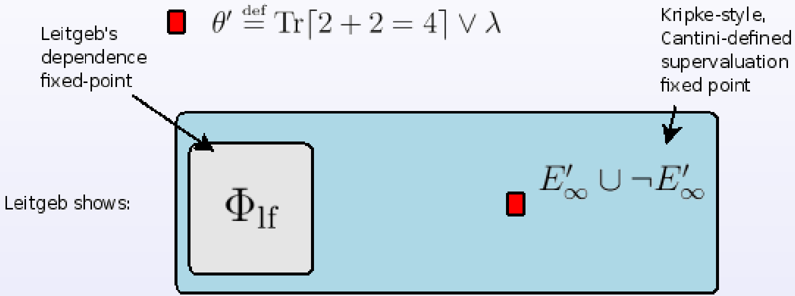

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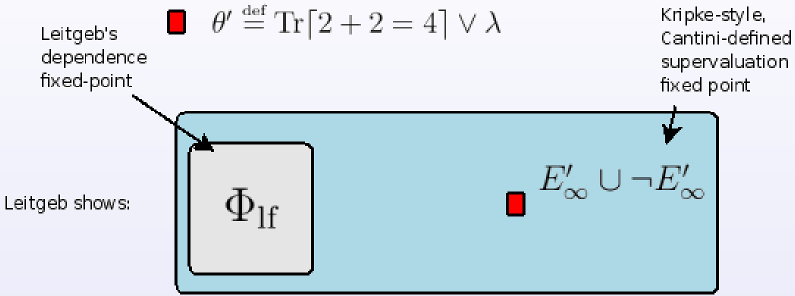

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    - $\Phi_0 = \emptyset, \Gamma_0 = \emptyset$ (the set of true sentences)
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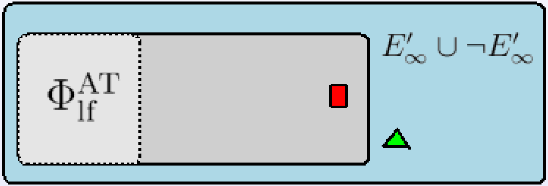

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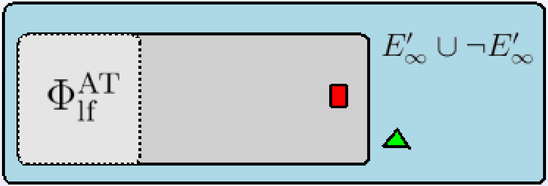

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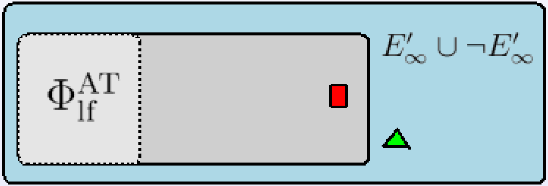

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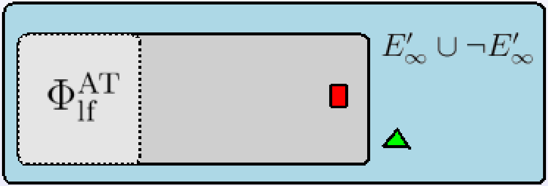

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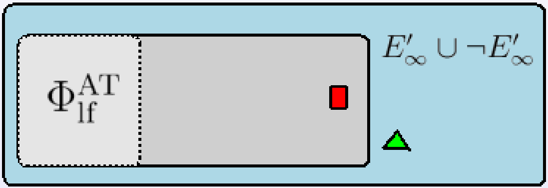

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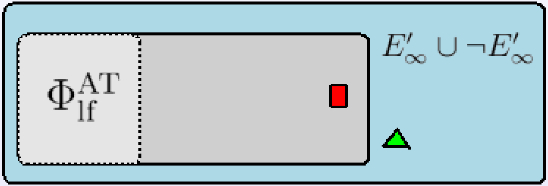

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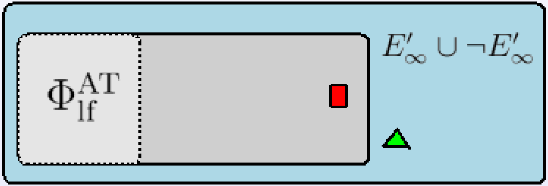

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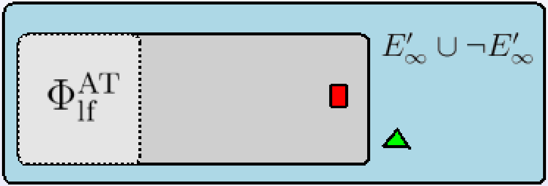

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  - Construction of $\Phi^{\mathrm{c,AT}}_{\mathsf{lf}}$ as before.

Overview of parameter changes

Conditional dependence with consistency requirement

equals Cantini

$$\Phi_{\mathrm{lf}}^{\mathrm{c,AT}}$$

Main result

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Main result

- Kripke (+classical) and Leitgeb (+conditional, +consistent) yield same grounded sentences.
- By changing parameters arrived at single notion of *groundedness*.

Equality proof: a sketch

Cantini=Leitgeb+consistency+conditionality

Every step in their constructions has become equal.

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3. The supervaluation operator FV is equal to Δ_c.

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Aboutness

- Whose notion responds best to pre–theoretic concept of *groundedness*?
- If there is a unique grounded set, then *groundedness* might actually derive from a much more general theory of *aboutness*.

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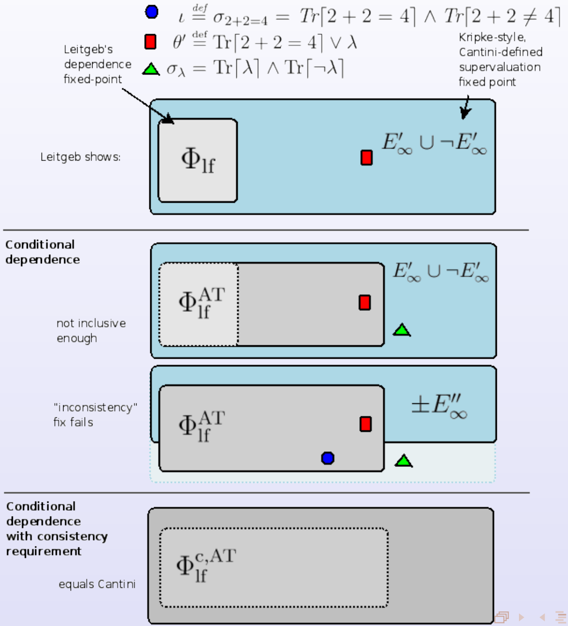
Grazie per l'attenzione

- Suggestions, critiques:
  Floris van Vugt, f.t.vanvugt@gmail.com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Leitgeb's dependence fixed-point
Kripke-style, Cantini-defined supervaluation fixed point
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Φ_{lf}
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Conditional dependence
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$\Phi^{\mathrm{AT}}_{\mathrm{lf}}$
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Dependencies

Simple dependence

ϕ is sensitive only to the Φ–sentences being true or not

Conditionality

ϕ is sensitive only to the Φ–sentences being true or not, but *presupposing* Σ–sentences are all true.

Conditional c–dependence

ϕ is sensitive only to the Φ–sentences being true or not, presupposing

- Σ–sentences true, and
- that the extension of Tr is consistent.

Kripke formally I

Given classical $\mathcal{L}$, $i_{\mathcal{L}}$ interpret $\mathcal{L}$ into a domain D.
Suppose $E \subset D$ (codes of) true $\mathcal{L}_{\mathrm{Tr}}$–sentences, and $A \subset D$ false sentences.

$$i_{\mathcal{L}_{\mathrm{Tr}}(E,A)}(\mathrm{Tr})(d) = \begin{cases} 1 & \text{if } d \in E \\ 0 & \text{if } d \in A \\ \uparrow & \text{otherwise} \end{cases} \tag{1}$$

and Kleene's strong three-valued logic.
Given $\mathcal{L}_{\mathrm{Tr}}(E, A)$ we can find

$$J_{(E,A)} \stackrel{\mathrm{def}}{=} \{\phi \in \mathcal{L}_{\mathrm{Tr}} | \phi \text{ is true under } i_{\mathcal{L}_{\mathrm{Tr}}(E,A)}\} \tag{2}$$

$$J^{-}_{(E,A)} \stackrel{\mathrm{def}}{=} \{\phi \in \mathcal{L}_{\mathrm{Tr}} | \phi \text{ is false under } i_{\mathcal{L}_{\mathrm{Tr}}(E,A)}\} \tag{3}$$

Given $E \subset \mathcal{L}_{\mathrm{Tr}}$ a "set of negatives" is defined: $\neg E \stackrel{\mathrm{def}}{=} \{\phi | \neg\phi \in E\}$. Since $\mathcal{L}_{\mathrm{Tr}}(E, A)$ is a closed language, we find that $J^{-}_{(E,A)} = \neg J_{(E,A)}$.

Kripke formally II

If we generalise the above procedure we find a sequence $(E_\alpha)_{\alpha \in \mathsf{On}}$ as follows:

- $E_0 = \emptyset$,
- $E_{\alpha+1} = J_{(E_\alpha, \neg E_\alpha)}$ and
- $E_\beta = \bigcup_{\alpha<\beta} E_\alpha$.

Monotonicity $\rightarrow$ fixed point E_∞.
A sentence ϕ of $\mathcal{L}_{\mathsf{Tr}}$ is defined to be *grounded* if it has a truth value (i.e. true or false) in $\mathcal{L}_{\mathsf{Tr}}(E_\infty, \neg E_\infty)$. Hence ϕ is grounded iff $\phi \in E_\infty \cup \neg E_\infty$.

Leitgeb formally

If $\phi \in \mathcal{L}_{\mathsf{Tr}}$ then $\mathsf{Val}_{\Psi}\phi$ denotes the truth value in the standard model of arithmetic enriched with a truth predicate which has extension $\Psi \subset \mathcal{L}_{\mathsf{Tr}}$. We define that ϕ *depends* on $\Phi \subset \mathcal{L}_{\mathsf{Tr}}$ iff for all $\Psi_1, \Psi_2 \subset \mathcal{L}_{\mathsf{Tr}}$, we have that if $\mathsf{Val}_{\Psi_1}\phi \neq \mathsf{Val}_{\Psi_2}\phi$ then $\Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$.
Then Leitgeb shows that $D_\phi \stackrel{\text{def}}{=} \{\Phi \subset \mathcal{L}_{\mathsf{Tr}} | \phi \text{ depends on } \Phi\}$ is a filter.
Similarly $D^{-1}(\Phi) \stackrel{\text{def}}{=} \{\phi \in \mathcal{L}_{\mathsf{Tr}} | \phi \text{ depends on } \Phi\}$. Leitgeb shows D^{-1} to be monotonic.
We define an ordinal sequence $(\Phi_\alpha)_{\alpha \in \mathsf{On}}$ as follows:

- $\Phi_0 = \emptyset$,
- $\Phi_{\alpha+1} = D^{-1}(\Phi_\alpha)$ and
- $\Phi_\beta = \bigcup_{\alpha<\beta} \Phi_\alpha$.

Least fixed point Φ_{lf} of *grounded* sentences.

Cantini

$\mathsf{Val}_\Psi \phi$ represents the truth value of the formula ϕ given that the Tr–predicate's extension is Ψ.
A set $\Psi \subset \mathcal{L}_{\mathsf{Tr}}$ will be considered *consistent* if, whenever $\psi \in \Psi$, then $\neg\psi \notin \Psi$.
Definition of FV, for all $\Phi \subset \mathcal{L}_{\mathsf{Tr}}$,
$\mathsf{FV}(\Phi) \stackrel{\text{def}}{=} \{\phi \in \mathcal{L}_{\mathsf{Tr}} | \forall \Psi \supset \Phi, \text{ s.t. } \Psi \text{ is consistent, } \mathsf{Val}_\Psi \phi = 1\}$,
Monotonous and consistency–preserving.
A sequence $(E'_\alpha)_{\alpha \in \mathsf{On}}$ is defined:

- $E'_0 = \emptyset$,
- $E'_{\alpha+1} = \mathsf{FV}(E'_\alpha)$ and
- $E'_\beta = \bigcup_{\alpha<\beta} E'_\alpha$. Its least fixed point is called E'_∞.

Conditional dependence formally (def. in Leitgeb[2005])

Conditional dependence

$\phi\, \mathsf{dep}_\Sigma(\Phi) \stackrel{\text{def}}{=}$ for all $\Psi_1, \Psi_2 \subset \mathcal{L}_{\mathsf{Tr}}$ s.t. $\Sigma \subset \Psi_1, \Psi_2$ it holds that $\mathsf{Val}_{\Psi_1}\phi \neq \mathsf{Val}_{\Psi_2}\phi \rightarrow \Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$

- $\Phi_0^{\mathsf{AT}} = \emptyset$,
  $\Gamma_0^{\mathsf{AT}} = \emptyset$,
- $\Phi_{\alpha+1}^{\mathsf{AT}} = \mathsf{D}^{-1}_{\Gamma_\alpha^{\mathsf{AT}}}(\Phi_\alpha^{\mathsf{AT}})$,
  $\Gamma_{\alpha+1}^{\mathsf{AT}} = \{\phi \in \Phi_{\alpha+1}^{\mathsf{AT}} | \mathsf{Val}_{\Gamma_\alpha^{\mathsf{AT}}}\phi = 1\}$,
- $\Phi_\beta^{\mathsf{AT}} = \bigcup_{\alpha<\beta} \Phi_\alpha^{\mathsf{AT}}$,
  $\Gamma_\beta^{\mathsf{AT}} = \bigcup_{\alpha<\beta} \Gamma_\alpha^{\mathsf{AT}}$,

Using that for all $\Phi, \Phi', \Sigma, \Sigma' \subset \mathcal{L}_{\mathsf{Tr}}$, for all $\alpha, \beta \in \mathsf{On}$,

1. If $\Phi \subset \Phi'$ and $\Sigma \subset \Sigma'$ then $\mathsf{D}^{-1}_\Sigma(\Phi) \subset \mathsf{D}^{-1}_{\Sigma'}(\Phi')$
2. (a) $\Phi_\alpha^{\mathsf{AT}} \subset \Phi_{\alpha+1}^{\mathsf{AT}}$ and (b) $\Gamma_\alpha^{\mathsf{AT}} \subset \Gamma_{\alpha+1}^{\mathsf{AT}}$

So a least fixed point, called $\Phi_{\mathsf{lf}}^{\mathsf{AT}}$.

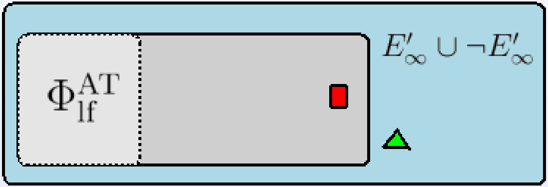
III: Consistency: removing from Cantini

$\theta' \stackrel{\text{def}}{=} \mathrm{Tr}\ulcorner 2+2=4 \urcorner \vee \lambda$

$\sigma_\lambda = \mathrm{Tr}\ulcorner \lambda \urcorner \wedge \mathrm{Tr}\ulcorner \neg\lambda \urcorner$

Conditional dependence

not inclusive enough



- **Problem** $\sigma_\lambda = \mathsf{Tr}\ulcorner \lambda \urcorner \wedge \mathsf{Tr}\ulcorner \neg\lambda \urcorner$
  - σ_λ false given any consistent Tr predicate: Cantini–grounded.
  - σ_λ depends on $\{\lambda, \neg\lambda\}$: not Leitgeb–conditional–grounded.

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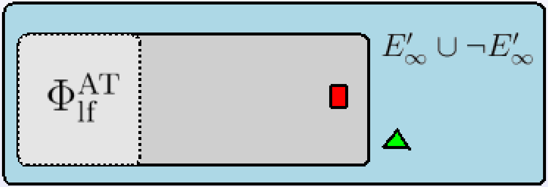

- **Problem** $\sigma_\lambda = \mathrm{Tr}\ulcorner \lambda \urcorner \wedge \mathrm{Tr}\ulcorner \neg\lambda \urcorner$
  - σ_λ false given any consistent Tr predicate: Cantini–grounded.
  - σ_λ depends on $\{\lambda, \neg\lambda\}$: not Leitgeb–conditional–grounded.
- **Solution** remove the consistency requirement in Cantini's FV.
  - $\mathrm{FV}'(\Phi) \stackrel{\text{def}}{=} \{\phi | \text{for } \textit{any } \Psi \supset \Phi, \mathrm{Val}_\Psi \phi = 1\}$
  - Thus obtained $\pm E''_\infty$ too exclusive: $\sigma_{2+2=4}$ becomes ungrounded.
    - $\sigma_{2+2=4}$ can be false in inconsistent Tr extending $\{2+2=4\}$: not Cantini'–grounded.
    - $\sigma_{2+2=4}$ depends on $\{2+2=4, 2+2\neq 4\}$: Leitgeb–grounded.

Conditional c–dependence formally

Conditional c–dependence

$\phi \, \mathrm{cdep}_{\Sigma}(\Phi) \stackrel{\text{def}}{=}$ for all *consistent*
$\Psi_1, \Psi_2 \supset \Sigma : \mathsf{Val}_{\Psi_1}\phi \neq \mathsf{Val}_{\Psi_2}\phi \to \Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$.

- $\Phi_0^{\mathrm{c,AT}} = \emptyset$,
  $\Gamma_0^{\mathrm{c,AT}} = \emptyset$,
- $\Phi_{\alpha+1}^{\mathrm{c,AT}} = \mathsf{D}^{-1}_{\mathrm{c},\Gamma_\alpha^{\mathrm{c,AT}}}(\Phi_\alpha^{\mathrm{c,AT}})$,
  $\Gamma_{\alpha+1}^{\mathrm{c,AT}} = \{\phi \in \Phi_{\alpha+1}^{\mathrm{c,AT}} | \mathsf{Val}_{\Gamma_\alpha^{\mathrm{c,AT}}}\phi = 1\}$,
- $\Phi_\beta^{\mathrm{c,AT}} = \bigcup_{\alpha<\beta}\Phi_\alpha^{\mathrm{c,AT}}$,
  $\Gamma_\beta^{\mathrm{c,AT}} = \bigcup_{\alpha<\beta}\Gamma_\alpha^{\mathrm{c,AT}}$

Reconciliation proof overview

- For all $\alpha \in \mathsf{On}$, $\Phi_\alpha^{\mathrm{c,AT}} = \pm\Gamma_\alpha^{\mathrm{c,AT}}$
- Redefinition
  - $\Gamma_0^{\mathrm{c,AT}} = \emptyset$,
  - $\Gamma_{\alpha+1}^{\mathrm{c,AT}} = \{\phi \in \mathsf{D}^{-1}_{\mathrm{c},\Gamma_\alpha^{\mathrm{c,AT}}}(\pm\Gamma_\alpha^{\mathrm{c,AT}}) | \mathsf{Val}_{\Gamma_\alpha^{\mathrm{c,AT}}}\phi = 1\} \stackrel{\mathsf{def}}{=} \Delta_c(\Gamma_\alpha^{\mathrm{c,AT}})$,
  - $\Gamma_\beta^{\mathrm{c,AT}} = \bigcup_{\alpha<\beta} \Gamma_\alpha^{\mathrm{c,AT}}$.
- $\phi \ \mathrm{cdep}_{\Phi}(\pm\Phi) \leftrightarrow \phi \in \pm\mathsf{FV}(\Phi)$
- For any consistent $\Phi \subset \mathcal{L}_{\mathsf{Tr}}$, $\Delta_c(\Phi) = \mathsf{FV}(\Phi)$
- For all $\alpha \in \mathsf{On}$, $\Phi_\alpha^{\mathrm{c,AT}} = \pm E'_\alpha$ and $\Gamma_\alpha^{\mathrm{c,AT}} = E'_\alpha$.

Tarski's definition of truth

Tarski

An infinite hierarchy of languages $(L_n)_{n\in\mathbb{N}}$ each of which includes a truth predicate Tr_n for the previous.

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Review

- Liar λ impossible to formulate.

Tarski's definition of truth

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An infinite hierarchy of languages $(L_n)_{n\in\mathbb{N}}$ each of which includes a truth predicate Tr_n for the previous.

Review

- Liar λ impossible to formulate.
- However, linguistically unsatisfying
  "Tr is *not one*, Tarski calls it many $(\mathrm{Tr}_n)_{n\in\mathbb{N}}$."

Only consistency – What if we skip conditionality? I

Consistency–dependence

$\phi\,\mathsf{dep}'(\Phi) \leftrightarrow \text{all consistent } \Psi_1, \Psi_2, \mathsf{Val}_{\Psi_1}\phi \neq \mathsf{Val}_{\Psi_2}\phi \rightarrow \Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$

Answer: same problem as before, $\theta = \mathsf{Tr}\ulcorner 2+2=4 \urcorner \vee \lambda$

Proposition $\theta\,\mathsf{dep}'(\Phi) \leftrightarrow \{2+2=4, \lambda\} \subset \Phi$

Proof Using $\mathsf{Val}_\Phi \theta = 1 \leftrightarrow \lambda \notin \Phi \vee 2+2=4 \in \Phi$.

- $\leftarrow$: Take any consistent Ψ_1, Ψ_2 s.t. $1 = \mathsf{Val}_{\Psi_1}\theta \neq \mathsf{Val}_{\Psi_2}\theta = 0$. Therefore $\lambda \notin \Psi_1 \vee 2+2=4 \in \Psi_1$ and $\lambda \in \Psi_2 \wedge 2+2=4 \notin \Psi_2$. Sufficient is to show $\Psi_1 \cap \{\lambda, 2+2=4\} \neq \Psi_2 \cap \{\lambda, 2+2=4\}$. Clearly $\Psi_2 \cap \{\lambda, 2+2=4\} = \{\lambda\}$ but it cannot be that $\Psi_1 \cap \{\lambda, 2+2=4\} = \{\lambda\}$ for we concluded $\lambda \notin \Psi_1 \vee 2+2=4 \in \Psi_1$.

Only consistency – What if we skip conditionality? II

Proposition (recall) $\theta\,\text{dep'}(\Phi) \leftrightarrow \{2+2=4, \lambda\} \subset \Phi$
Proof part II

- $\rightarrow$: suppose the $\theta\,\text{dep'}(\Phi)$ but $\{2+2=4, \lambda\} \not\subset \Phi$. One of the following must be true:
  - $\lambda \notin \Phi$. Clearly $1 = \text{Val}_{\emptyset}\theta \neq \text{Val}_{\{\lambda\}}\theta = 0$. Because $\theta\,\text{dep'}(\Phi)$ it would follow that $\emptyset \cap \Phi \neq \{\lambda\} \cap \Phi = \emptyset$, contradiction.
  - $2+2=4 \notin \Phi$. Now $0 = \text{Val}_{\{\lambda\}}\theta \neq \text{Val}_{\{\lambda,2+2=4\}}\theta = 1$. Because $\theta\,\text{dep'}(\Phi)$ this means $\{\lambda\} \cap \Phi \neq \{\lambda, 2+2=4\} \cap \Phi = \{\lambda\} \cap \Phi$, absurd.

Assumptions It has been assumed θ is consistent with itself and with $2+2=4$.