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On Axiomatic Principles in the Debate on the Circular Justification of Induction

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ABSTRACT

Some inductivists argue that although induction cannot be justified without initially assuming it to be true, the same holds for deduction. However, in this essay it will be argued that deduction should be considered an axiom of our reasoning and it is questionable whether induction can be axiom as well. Some concepts related to the subject matter will be introduced and explained, such as justification-affording arguments, knowledge and justification.

1. INTRODUCTION

In his recent publication *The Empirical Stance*, Bas van Fraassen argues that the principle of empiricism, which says that all knowledge in the end is based on empirical observation, cannot be maintained in its current form. For if all knowledge is based on empirical experience, and this principle apparently is not, then it would be self-contradictory to try to hold it at any cost.
However, empiricists are allowed to act according to it, without having to justify it, van Fraassen argues in this recent publication. Such a principle could serve as a 'stance,' a certain attitude towards philosophy or science.
Such a 'stance' has several characteristics in common with axioms, for instance that they are not questioned and that their validity determines the validity of the statements which are based on them.
Using this as a starting point, this paper will consider a more fundamental issue in the philosophy of science: the problem of the justification of induction. In the first part of this essay some important concepts will be introduced and clarified;

knowledge, justification, circular arguments, viciousness, justification-affording arguments and several notions related to them. The second part of this essay will deal with the case of the justification of induction and will apply the theories that have been discussed.
Finally, the thesis that deduction should be considered an axiom will be defended, and the question whether or not induction should as well, will be addressed. Perhaps deduction will be the rationalist's stance, and induction the inductivist's.

2. CONCEPTS CONCERNING KNOWLEDGE

Before a more detailed discussion of the subject matter will be possible, a definition of several concepts is indispensable, starting with the notion of knowledge. According to the classical definition, the following holds:

A knows that P if and only if;
(i) A believes P to be true
(ii) P is true
(iii) A is justified to believe that P

Accordingly, we can paraphrase knowledge as a justified true belief.

In the following elaboration I will restrict my analysis to simple sentences of the form "α is φ", where α denotes the syntactical subject of the sentence, and φ the predicate that is matched to it.
Several kinds of knowledge exist and the distinction between them is of importance to this discussion. The 17th Century philosopher Immanuel Kant

introduced the pairs of terms analytic/synthetic to nuance the difference between the existing terms a priori/a posteriori.

In his *Prolegomena*, the preface to his famous *Kritik der Reinen Vernunft*, Immanuel Kant writes (A25):

> Analytische Urteile sagen im Prädikate nichts als das, was im Begriffe des Subjekts schon wirklich, obgleich nicht so klar und mit gleichem Bewußtsein gedacht war. Wenn ich sage: Alle Körper sind ausgedehnt, so habe ich meinen Begriff vom Körper nicht im mindesten erweitert, sondern ihn nur aufgelöset, indem die Ausdehnung von jenem Begriffe schon vor dem Urteile, obgleich nicht ausdrücklich gesagt, dennoch wirklich gedacht war; das Urteil ist also analytisch.

According to Kant, an analytic statement is a statement whose subject (α) is contained within its predicate (φ). Furthermore, its negation is self-contradictory (referred to by Kant as the Law of Contradiction). Nowadays, the following definition is more popular: analytic statements are true solely by virtue of the meanings of the terms it employs. For instance, Kant uses the sentence "All bodies are extended," to illustrate this. It is analytic, because the definition of a body includes that it is extended.

A synthetic statement is a statement which is not analytic, therefore, the truth of synthetic statements cannot be assessed by simply breaking the concepts that are used up in their semantic parts. In other words, in a synthetic statement, the subject (α) and the predicate (φ) are not defined in such a way that they evidently match.

Kant introduced this distinction to nuance the existing difference between a priori and a posteriori judgements. Originally, a priori judgements meant only what the translation literally means: from what goes before; from cause to effect. In Kant's days, a priori is used to refer to propositions that are known

independently of empirical evidence. For instance, according to this definition, mathematical propositions are to be considered a priori, because they are not based on empirical evidence. The proposition E_1 illustrates this.

(E_1) $\underbrace{\text{“the sum of a triangle's angles}}_{\alpha}$ is $\underbrace{\text{180º”}}_{\varphi}$

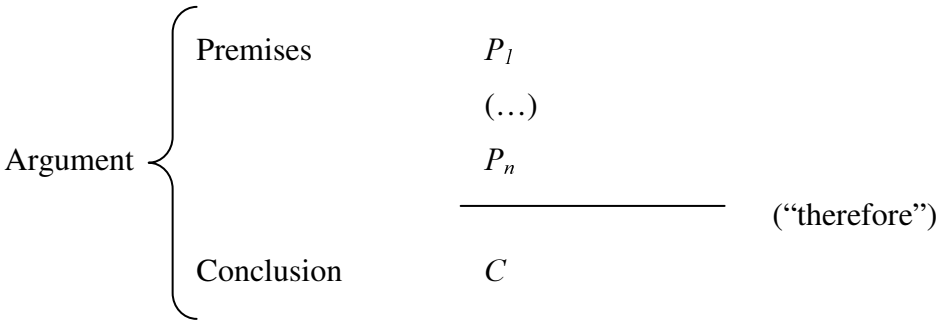
Kant argued that all a posteriori propositions are synthetic. It would be relatively useless to test a great number of bodies and then (inductively) infer that all bodies are extended. In such a case one would utter an analytic a posteriori statement, but it is relatively trivial. By contrast, according to Kant, a priori concepts can be both analytic and synthetic. I will consider the a priori statement E_1 synthetic, because the *definition* of the subject (α) does not include the predicate (φ) “is always 180º”. The law E_1 happens to be true in any thinkable case, as can be shown by deduction.

This difference between the kinds of knowledge also implies a difference between the possible ways of their justification. For instance, a priori statements will not rely on empirical evidence to be justified, whereas a posteriori statements typically do.

3. CONCEPTS CONCERNING CIRCULARITY

In the following discussion the concept of argument will be referred to several times. Therefore it will be wise to introduce the following scheme. An argument consists of any finite *n* number of premises and a conclusion that is drawn on the basis of the premises only, as is illustrated below:

Figure 1: The definition of an argument



Moreover, a target audience of the argument will be referred to as A.

Vicious circularity is another important concept that will be used throughout this essay.

The Oxford Companion to Philosophy describes vicious circularity as follows:

> [Vicious circularity is a]n argument assuming its conclusion as a premiss (begging the question), or a definition of an expression in terms of itself. Russell argued that paradoxes in the foundations of mathematics - for example, his paradox of the class of all classes that are not members of themselves - depend on a kind of vicious circularity, violating the maxim 'Whatever involves all of a collection must not be one of the collection'.

A few remarks that will be made later on will be applicable to the Russell criticism of mathematics as well.

In his recently published article *Justification-Affording Circular Arguments*, Andrew D. Cling argues also introduces the concept justification-affording argument. According to him, an argument "is justification-affording for

audience *A* just in case, given *A*'s epistemic predicament, *A* would acquire justification for believing C by reasoning to C through P" (252-253). Moreover, Cling argues that "[a]ccording to this account [an argument] is justification-affording for *A* if, and only if, *A*'s reasoning to C through P is an essential part of a sufficient condition of *A*'s acquiring justification for believing C" (253). First of all, let us consider an example.

P_1	Zero is an even number.
P_2	Adding an even number to an even number results in an even number.
C	A sum of any amount of even numbers results in an even number.

This argument can be justification-affording, because assuming the premises to be true, the conclusion is true as well. Therefore, imagine that the premises are already accepted in a community (i.e. they are justified) then the conclusion is justified as well.
Second of all, one should note that this accounts for the triviality in the nature of arguments of the form *Q, therefore Q*. For example: "A man was killed, therefore a man was killed." These arguments can never be justification-affording, because reasoning through Q to Q is not a sufficient condition for acquiring justification to believe Q, for obvious reasons.

Additionally, Cling distinguishes several functions an argument could have with respect to its target audience *A*:

i. logical displaying consequences of P
ii. persuasive producing belief in C

iii. epistemic providing justification-affording evidence for C
iv. explanatory providing an account of why or how C is true

He writes, that "we may […] evaluate an argument by considering not only whether it is valid or sound but also whether it can persuade the target audience" (253).

Also, Cling distinguishes between several types of circularity in arguments. His theory will be presented here in a tabular form, which corresponds to '*Y* is a necessary condition for *X*', where both variables will be replaced with an expression: *Q* (which means: *the truth of Q*), B(*Q*) (which means: *A believes Q*) or J(*Q*) (which means: *A justifiably believes Q*).

X
(target)

Y (condition)		*P*	B(*P*)	J(*P*)
	C	(example)		
	B(*C*)			
	J(*C*)		Doxastic	Epistemic
	Bottom Tier			

For instance, the cell that reads (example) corresponds to the case in which the conclusion is a necessary condition for the premise to be true.

At first sight it may seem that an argument containing a premise that depends on the conclusion is epistemically problematic. Cling shows it is not, because, he argues, it is just for premises to entail the conclusion, although it results in this kind of circularity.

For instance, this is the case with the famous Modus Ponens:

P_1	If p then q
P_2	p
C	q

If the conclusion is false (which occurs whenever q is false), then the premises cannot both be true. Therefore, Cling argues, the conclusion is a necessary condition for the premises to be true. However, obviously, in this case it is not problematic. In fact, any type of circularity in which the condition does not include justifiably believing the conclusion, is not problematic (Cling 254). Potential problems arise if we cannot justify a conclusion C without assuming it to be justified already. Arguments of this kind are represented in the bottom row of the table, which from now on will be referred to as the Bottom Tier. Cling goes on to argue that the Bottom Tier situation in which the premise depends upon justifiably believing the conclusion is not problematic either, because the sentence "*I am a conscious human being therefore I have a brain*" involves this kind of dependency, but can have "probative force" (Cling 255). Therefore, the potentially problematic situations can be narrowed to *doxastically* circular and *epistemically* circular arguments.

Descartes' famous cogito *I think, therefore I am* is an example of a doxastically circular argument, because it seems that one has to believe that one exists in order to accept that one thinks. This renders the conclusion that a doxastically circular argument cannot persuade someone who doubts inevitable, but it can be justification-affording for the audience that already believes the truth of the conclusion. That would be the case if we have evidence for believing that *I think* that does not include *I am*.

Cling deals with the question whether epistemically circular arguments can be not viciously circular.
One argument that would provide a negative answer to this question is:

i. An argument *P therefore C* can be justification-affording for *A* only if *A* already justifiably believes *P*.
ii. In an epistemically justifiable argument, the conclusion is one of the premises.

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iii. Therefore, epistemically circular arguments are vicious.

However, Cling analyses, that this argument assumes all justification-affording arguments to be justification-creating. He argues that epistemically circular arguments can be justification-affording in the sense that they enhance *A*'s justification for believing *C*. This would be the case when a premise is supported by a conclusion that, together with other premises, provides additional evidence for the conclusion, because although such an argument could not serve purpose (ii) (which is, to persuade an audience *A* that *C* is true) with respect to an audience who is not yet persuaded, it could serve all other three purposes mentioned by Cling.

This section will be closed with some remarks on the concept of axiom.
The Oxford Companion to Philosophy provides the following account of how the axiom is defined.

> An axiom is one of a select set of propositions, presumed true by a system of logic or a theory, from which all other propositions which the system or theory endorses as true are deducible - these derived

> propositions being called theorems of the system or theory. Thus, Pythagoras' theorem is deducible from the axioms of Euclidean geometry. The axioms and theorems of a system of logic - for instance, of the propositional calculus - are regarded as being true of logical necessity.

Firstly, by their very nature, all axioms will be considered analytic statements since an axiom can be seen as a definition. For instance, the axiom that all right angles are equal is true per definition and therefore analytic.

Second, axioms provide foundation for all theorems that are deduced from them. Therefore, the truth of axioms determines the truth of these theorems. For instance, the geometry that Euclid proposed in the antiquity has, for a long time, been used without question in most sciences, such as Newtonian mechanics. However, both Quantum Mechanics and Einstein's General Relativity theory required the deployment of a completely different geometry. However, these new theories did not imply that the theorems proposed by Euclidian geometry were incorrect, given that the axioms were true. That is to say that the rules of inference were correctly applied and Euclidian geometry is still considered a coherent theory. The criticism of these new theories focused on the axioms that Euclidian geometry was based on and simply denied their applicability to reality. General Relativity explicitly predicts that space can disobey several of Euclidian geometry's axioms (*Oxford Companion to Philosophy*, "philosophical problems of physics", par. 8). An immediate consequence was, that if those critics were correct, none of Euclid's theorems, nor any others derived from his axiomatic system could be correctly considered a valid description of reality without additional justification.

Therefore, mathematical or logical theories can only be usefully applied once one acknowledges the truth of their premises, and cannot be denied once one does acknowledge that truth of the premises.

Using the theories introduced so far, a major problem related to circularity and justification will be addressed in the following sections, namely the problem of induction. I will argue that in this case, some of the problems can be avoided by granting certain principles the status of axiom.

4. PROBLEM OF INDUCTION AND DEDUCTION

Induction is reasoning from a specified number of particular cases to a general rule. There are several types of induction:

Induction type	Reasoning
induction to a particular (IP)	All observed As are Bs. The next A will be a B.
statistical syllogism	Many observed As are Bs. The next A will be a B.
arguments from analogy	Several individuals with properties R, S and T had property U. An individual with properties R, S and T will have property U.

A few comments will have to be made on these notions.
First of all, some philosophers defend a weaker form of induction, which concludes that *probably* the next A will be a B. As will be discussed later on, this does avoid some of the problems that the defenders of the strong version of induction face.

Secondly, this essay will provide an account of the behaviour of IP in particular. However, the discussion can equally correct be applied to the other forms of induction as well, since they all rely on the underlying inductive inference of reasoning from a limited number of particular cases to a general rule.
The problem of induction will be explained in detail later in this essay, and can be divided into two separate problems, according to the philosopher Peter Lipton: the problem of *description* and the problem of *justification*. The problem of description deals with the rules that should be formulated to the making of valid inductive inferences. The problem of justification deals with how the inductive inference rule can be justified. In other words: how can we be justified in inferring a general rule from a specified number of observed occurrences?
This essay will deal only with the problem of justification.

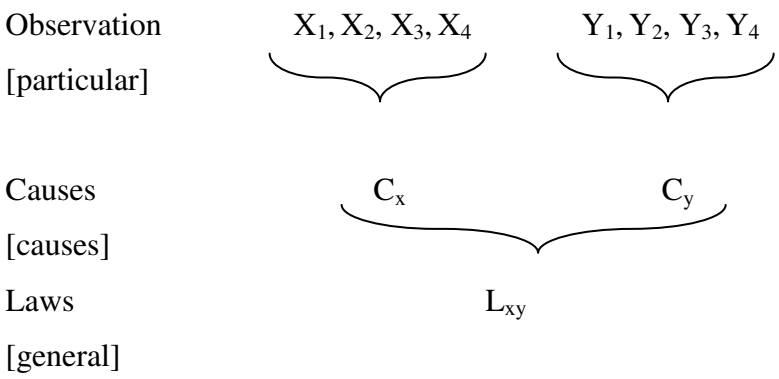
The problem of the justification of induction has been thought about since the antiquity. Aristotle argued that the human brain possessed a faculty that could flawlessly see the essence of things on the basis of observation only and that, bluntly put, induction requires no justification because it is based on intuition. However, the 18th century British philosopher David Hume argued that even though induction can be justified psychologically, it lacks any epistemic justification.
Hume became famous with his refutation of our perception of causality, arguing that causal relationships cannot be perceived but are without justification inferred from observations by our mind. If one sees a ball hitting another ball, one would infer that one is justified in saying that the second ball started moving, because the first ball hit it. However, Hume argues that this is

in no way justified. The only thing we see is the first ball rolling and then the other ball rolling. The causal relationship between the two is what our brain produces to make sense of the events. But we do not really *see* the causal relationship.

In the light of this argument Hume's objections against induction can be understood. James Ladyman writes in his book *Understanding Philosophy of Science*, that "Hume observes that our inductive practices are founded on the relation of cause and effect, but when he analyses this relation he finds that all that is, from an empiricist point of view, is the constant conjunction of events, in other words, the objective content of a posited causal relation is always merely that some regularity or pattern in the behaviour of things holds" (40). Hume himself writes in An Enquiry Concerning Human Understanding (71):

> It may only, perhaps, be pretended that the mind can perceive, in the operations of matter, some farther connexion between the cause and effect; and connexion that has not place in voluntary actions of intelligent beings. Now whether it be so or not, can only appear upon examination; and it is incumbent on these philosophers to make good their assertion, by defining or describing that necessity, and pointing it out to us in the operations of material causes.

As is depicted below, we infer a causal relationship from several X_n observations that accounts for a certain law to be stated. For instance, when we see a ball hitting another, perhaps several times, then we might agree that there is a causal relationship, from which we will infer the law that every ball that hits another one will roll. However, since Hume argues that inferring causes from observations is not justified, the laws that follow them are not justified either.



For instance, many people agree that every day the sun will rise. Many even consider it unthinkable that it will not rise one day. But we have inferred that the sun will rise *every* day on the basis of observations on only a limited number of days on which we have observed that the sun rose. In order to be absolutely certain that the sun rises every day, one should

(1) observe *every* day whether the sun will rise, and

(2) be able to see some other relation between the meaning of the terms 'sun' and 'day' or between the things they refer to. And that relation must then be able account for their generally postulated constant conjunction, formulated in the law.

As for (1), obviously this is impossible and would take away the predictive force of scientific laws. As for (2), many philosophers argue that such a 'relation' cannot be perceived.

Albert Casullo points out the following in his article *The Coherence of Empiricism* (32):

> For if the conclusions of the inferences genuinely go beyond the content of direct experience, then it is impossible that those inferences could be entirely justified by appeal to that same experience.

In other words, inferring some super-empirical concepts (such as causality) on the basis of observation alone, cannot be justified by that observation alone. This issue will be discussed further later on.

Hume's argument can be summarised as follows (Curd and Cover 498):

P_1	If IP can be shown to be justified, then there is an argument that shows it.
P_2	Arguments are either deductively valid or inductive.
P_3	No deductively valid argument can justify IP, because of underdetermination.
P_4	No inductive argument can justify IP, because that would be circular.
C	IP cannot be shown to be justified.

P_3 refers to the logical possibility that a regularity that is perceived in the past, will fail to hold in the future. For instance, one might make the following inductive inference: *All observed As have been B, therefore, all As are B*. However, it is perfectly logically thinkable that not all *A*s are *B*s. For instance, we could have accidentally stumbled upon only the *A*s that are *B*s, whereas there are perhaps many *A*'s that are *B*'s we did not see.
This statement evoked several philosophical counterarguments. This discussion will focus on only some of them.
First an answer will be discussed that refers back to the concepts that were introduced in section 2. More or less simplified, it claims that some synthetic truths can be known a priori. Hume thought that all synthetic judgements were a posteriori and he called them matters of fact. The philosopher Immanuel Kant first introduced the notion that a priori judgements could be synthetic in connection with his nuance between a priori and a posteriori judgements. Kant argued that some principles can be known a priori, because they are determined

by the way our minds work. In order to perceive reality, the raw sense data that is collected by our various senses has to be combined and processed by what Kant called the categories of the mind and causality is one of them. Kant agrees with Hume that causality is imposed on the sense data by the human brain, so we cannot know whether or not the causality is actually present in reality. But because our brain shapes any sense data, it is no use talking about a reality independent of our human brain, because we cannot perceive such a reality independently.

In brief, causality is a structure that is imposed on reality by our brain, and therefore reliable. Therefore, principles such as causality are synthetic a priori, their existence or the nature of their inner mechanics is not given by definition, but we can know they are there because of the way our mind works.

However, nowadays, this claim of Kant's is considered too optimistic, since many of the laws he thought were synthetic a priori and beyond doubt, such as Newtonian mechanics and the Euclidian geometry as a valid description of space are, as indicated before, proven not to be correct in every situation.

5. AXIOMATICAL STATUS IN THE INDUCTION DEBATE

The second counter-argument is considered more worthwhile. It disagrees with P_4 in Hume's argument, arguing that a premise in the justification of induction can be based on induction. There are two reasons given for this argument, which will be discussed here.

Firstly, it is stated that justifying induction by presupposing induction, is circular, but not vicious, because it is *rule* circular and not *premise* circular. Recall from the introduction that an argument of the kind which presupposes

the conclusion would be epistemically circular and would look like the following:

P_1 Induction is justified.
P_2 In the past induction has worked.

C Induction is justified.

It is epistemically circular, because in order to justifiably believe the conclusion C,

(1) the premises must be justifiably believed (a condition to make any argument justification-affording), and
(2) the first premise presupposes the conclusion to be true.

Therefore, according to Cling, such an argument cannot be justification-affording, but only justification-enhancing.
However, inductivists argue that P_1 is not actually a premise, but rather the rule that is being used to infer C through the premises P. This issue will be further elaborated on later.

Secondly, a more controversial argument has been brought forward by Lewis Carroll (1895). It states that even deduction, which is considered perfectly legal and justified, cannot be justified without first assuming it to be justified. Ladyman paraphrases the argument in his book *Understanding Philosophy of Science* (49):

> [C]onsider the following pattern of deductive inference: someone believes some proposition, p, and they also believe that if p is true then another proposition q follows, and so they infer q. What could you say to someone who refused to accept this form of inference? […] [You

> might say:] look, you believe *p*, and you believe if *p* then *q* is true then *q* must be true as well. They reply, 'Okay, I believe *p*, and I believe if *p* then *q*, and I even believe that if *p* is true and if *p* then *q* is true then *q* must be true as well; however, I don't believe *q*'. What can we say now?

This makes inductivists claim that their assuming induction to be justified completely valid, even though only a circular argument can justify induction, the same thing holds for deduction. However, before will be elaborated on this argument, it should be pointed out that this argument does not solve the problem of induction, but merely tries to evade the duty of justification of induction.

Now there are two problems: induction still does not seem to be justified after all and deduction is on the slope as well.
Both can potentially be solved if we allow certain statements the status of axiom, which is the attitude that will be advocated here.
First of all the essence of the deductive inference will be discussed here.
Consider the following formulation of the Modus Ponens:

Modus Ponens

P_1	If *p* then *q*
P_2	*p*
C	*q*

Reflecting only upon its very nature it is evident that this argument is analytic, because the words *If* and *then* are defined in such a way that once they are combined in this manner, the premises entail the conclusion.
Even although I agree that one cannot convince somebody who does not accept deduction that deduction is valid, that is not problematic. Deduction is a reasonable candidate to be awarded the status of axiom. By its very nature, deduction

(1) cannot be proven without epistemic circularity
(2) is analytic, therefore
(3) denying its truth seems self-contradictory

The first characteristic applies to mathematical axioms, as do the other two, as was concluded from the discussion of concepts earlier in this essay.
For instance: all right angles are equal to each other. This is analytic in the sense that we define right angles to have one certain angle and therefore two angles who are both right, will be equal. It cannot be proven without epistemic circularity, therefore it is an axiom.

Inductivists might attempt to argue that induction could then be considered an axiom as well. Once a principle of induction could be accepted as an axiom, inductive arguments could be justified by a *principle of induction* P_i, as is proposed by several philosophers:

P_i	Induction is justified.
P_1	*n* observed *A*s have been *B*s.
C	All As are B.

Moreover, I would argue that induction is unconsciously considered an axiom by many scientists, who view doubting induction as irrational.
However, this is perhaps not as legal as it is with deduction. Several remarks can be made as to this axiomatic induction.
Firstly, induction does not share deduction's analytic nature. Therefore, to deny the conclusion of an inductively valid argument is not a self-contradiction (as is the case with a deductive argument). Possibly, one could argue that every axiom would have to be analytic in nature. However, dealing with such arguments would go beyond the scope of this paper.
Secondly, as argued for in section 3, the truth of the statements that are made based on an axiom depends on the truth of the axiom. Therefore the truth of the scientific knowledge that is gathered based on an induction axiom depends on the truth of that axiom, which is a sacrifice that will have to be made.

6. CONCLUSION

In this essay it is argued that perhaps deduction should be granted the status of an axiom, and the question whether induction could as well is commented on briefly.
Before rounding of this essay, it should be pointed out that this claim does not solve the problem of induction. The axiomatic status of deduction is introduced to answer to the argument that deduction cannot be justified without circularity. If the philosophical community would indeed grant deduction the status of axiom, then

(1) a decision as to be made as well as to whether induction can be an axiom as well, and
(2) the question remains whether the axiom is correct; as well as discussion is possible about the question whether Euclidian axioms can be applied

to reality, the question whether these newly defined axioms can has to be answered as well.

Either way, no definitive answers on this matter are given here or will be given in the near future.

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