# Paradoxes and Self–Reference

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# Truth

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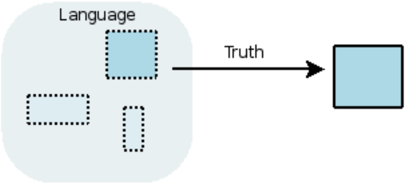


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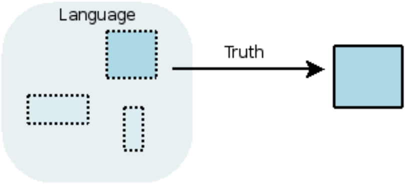


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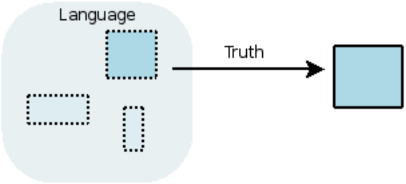


Truth models relationship between language and external world.

## Concept of "truth"

- Intuitively clear, yet
- Immediately problematic

# Paradox and the concept of truth

## Truth–predicate

A predicate Tr that applies to any sentence $\phi$, such that (*T–equivalence*)

$$\mathrm{Tr}\ulcorner\phi\urcorner \text{ if and only if } \phi$$

e.g. "⌜snow is white⌝ is true" iff show is white.

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Is it true or false?

- $\mathrm{Tr}\ulcorner\lambda\urcorner \rightarrow \lambda \rightarrow \neg\mathrm{Tr}\ulcorner\lambda\urcorner$
- $\neg\mathrm{Tr}\ulcorner\lambda\urcorner \rightarrow \neg\lambda \leftrightarrow \mathrm{Tr}\ulcorner\lambda\urcorner$

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## Groundedness

Referring (in)directly to non–semantic states of affairs.

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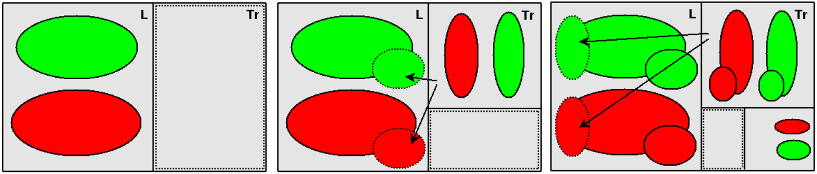

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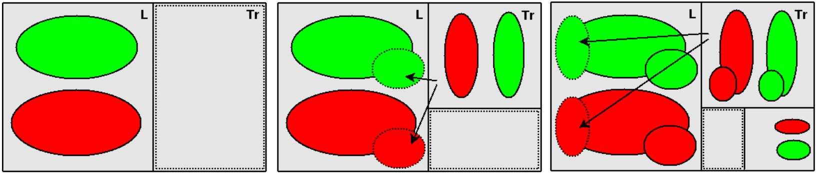


*Grounded*: eventually attributed true or false

# Kripke's definition of truth

**Kripke**

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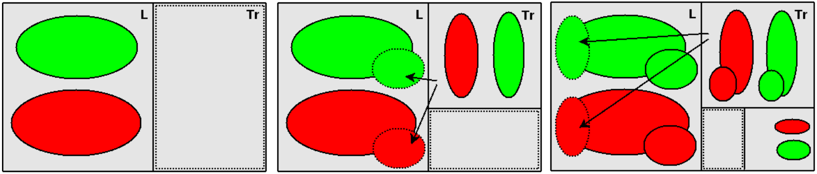
- All T–equivalences hold, but $\mathsf{Val}(\lambda) = \mathsf{n}$.

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- T–equivalences are required to hold only for grounded sentences.

## Main question: comparison of “groundedness”

- Kripke and Leitgeb: constructions very similar, but not the same set of grounded sentences.

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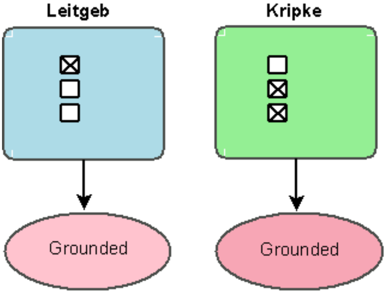
## Hypothesis

There is one notion of groundedness, but Kripke and Leitgeb's *parameter settings* differ.



# I: Classical or three–valued logic

- **Problem** $\mathrm{Tr}\ulcorner\lambda\urcorner \vee \neg\mathrm{Tr}\ulcorner\lambda\urcorner$

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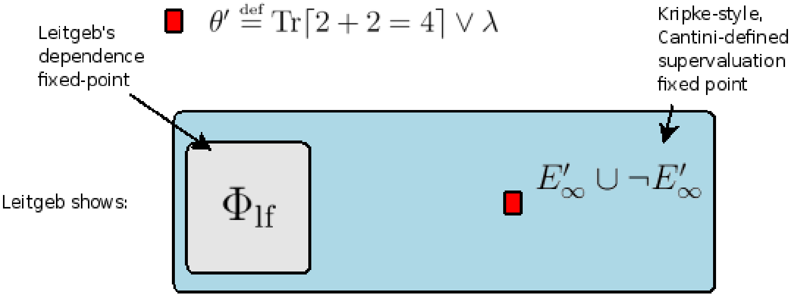

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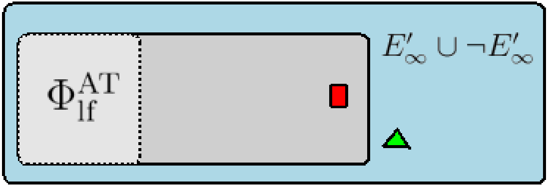
$$\theta' \stackrel{\text{def}}{=} \mathrm{Tr}\ulcorner 2+2=4 \urcorner \vee \lambda$$

$$\sigma_\lambda = \mathrm{Tr}\ulcorner \lambda \urcorner \wedge \mathrm{Tr}\ulcorner \neg\lambda \urcorner$$

**Conditional dependence**

not inclusive enough



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**Conditional dependence with consistency requirement**

equals Cantini

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- **Main result** Kripke (+classical) and Leitgeb (+conditional, +consistent) yield same grounded sentences.

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- Paradoxes are problematic for the definition of truth
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- *"Grounded is one, Cantini and Leitgeb call it different names."*

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## Overview

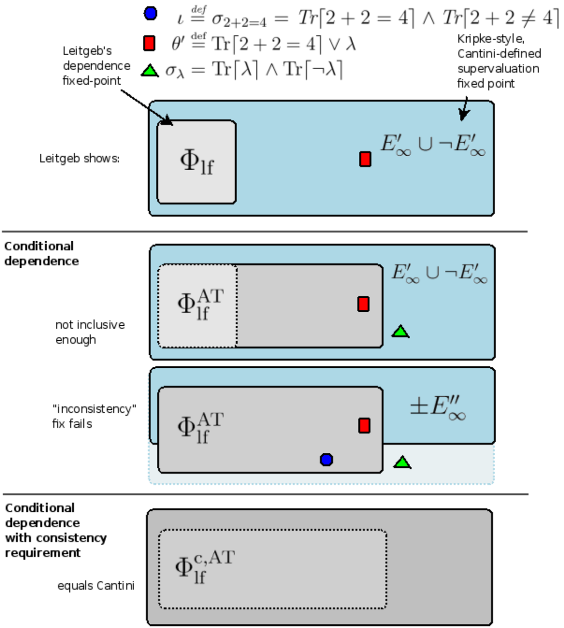
- Paradoxes are problematic for the definition of truth
- Kripke (Cantini) and Leitgeb keep equivalences $\mathrm{Tr}\ulcorner\phi\urcorner \leftrightarrow \phi$ for "grounded sentences"
- *"Grounded is one, Cantini and Leitgeb call it different names."*

## Perspectives

- *Aboutness* to generalise "dependence"

## Grazie per l'attenzione

- *Se non è sul web, non esiste.*
- Therefore, you can find the paper online:
  - google "Floris van Vugt", or
  - http://vanvugt.cjb.net/


$\iota \stackrel{def}{=} \sigma_{2+2=4} = Tr[2+2=4] \wedge Tr[2+2\neq 4]$
$\theta' \stackrel{\mathrm{def}}{=} \mathrm{Tr}[2+2=4] \vee \lambda$
$\sigma_\lambda = \mathrm{Tr}[\lambda] \wedge \mathrm{Tr}[\neg\lambda]$
Leitgeb's dependence fixed-point
Kripke-style, Cantini-defined supervaluation fixed point
Leitgeb shows:
$\Phi_{\mathrm{lf}}$
$E'_\infty \cup \neg E'_\infty$
Conditional dependence
not inclusive enough
$\Phi_{\mathrm{lf}}^{\mathrm{AT}}$
$E'_\infty \cup \neg E'_\infty$
"inconsistency" fix fails
$\Phi_{\mathrm{lf}}^{\mathrm{AT}}$
$\pm E''_\infty$
Conditional dependence with consistency requirement
equals Cantini
$\Phi_{\mathrm{lf}}^{\mathrm{c,AT}}$

# Dependencies

**Simple dependence**

$\phi$ is sensitive only to the $\Phi$–sentences being true or not

**Conditionality**

$\phi$ is sensitive only to the $\Phi$–sentences being true or not, but *presupposing* $\Sigma$–sentences are all true.

**Conditional c–dependence**

$\phi$ is sensitive only to the $\Phi$–sentences being true or not, presupposing

- $\Sigma$–sentences true, and
- that the extension of Tr is consistent.

# Kripke formally I

Given classical $\mathcal{L}$, $i_{\mathcal{L}}$ interpret $\mathcal{L}$ into a domain $D$.
Suppose $E \subset D$ (codes of) true $\mathcal{L}_{\mathrm{Tr}}$–sentences, and $A \subset D$ false sentences.

$$i_{\mathcal{L}_{\mathrm{Tr}}(E,A)}(\mathrm{Tr})(d) = \begin{cases} 1 & \text{if } d \in E \\ 0 & \text{if } d \in A \\ \uparrow & \text{otherwise} \end{cases} \tag{1}$$

and Kleene's strong three-valued logic.
Given $\mathcal{L}_{\mathrm{Tr}}(E, A)$ we can find

$$J_{(E,A)} \stackrel{\text{def}}{=} \{\phi \in \mathcal{L}_{\mathrm{Tr}} | \phi \text{ is true under } i_{\mathcal{L}_{\mathrm{Tr}}(E,A)}\} \tag{2}$$

$$J^{-}_{(E,A)} \stackrel{\text{def}}{=} \{\phi \in \mathcal{L}_{\mathrm{Tr}} | \phi \text{ is false under } i_{\mathcal{L}_{\mathrm{Tr}}(E,A)}\} \tag{3}$$

Given $E \subset \mathcal{L}_{\mathrm{Tr}}$ a "set of negatives" is defined: $\neg E \stackrel{\text{def}}{=} \{\phi | \neg\phi \in E\}$.
Since $\mathcal{L}_{\mathrm{Tr}}(E, A)$ is a closed language, we find that $J^{-}_{(E,A)} = \neg J_{(E,A)}$.

# Kripke formally II

If we generalise the above procedure we find a sequence $(E_\alpha)_{\alpha \in \mathsf{On}}$ as follows:

- $E_0 = \emptyset$,
- $E_{\alpha+1} = J_{(E_\alpha, \neg E_\alpha)}$ and
- $E_\beta = \bigcup_{\alpha<\beta} E_\alpha$.

Monotonicity $\longrightarrow$ fixed point $E_\infty$.

A sentence $\phi$ of $\mathcal{L}_{\mathsf{Tr}}$ is defined to be *grounded* if it has a truth value (i.e. true or false) in $\mathcal{L}_{\mathsf{Tr}}(E_\infty, \neg E_\infty)$. Hence $\phi$ is grounded iff $\phi \in E_\infty \cup \neg E_\infty$.

# Leitgeb formally

If $\phi \in \mathcal{L}_{\mathsf{Tr}}$ then $\mathsf{Val}_{\Psi}(\phi)$ denotes the truth value in the standard model of arithmetic enriched with a truth predicate which has extension $\Psi \subset \mathcal{L}_{\mathsf{Tr}}$.

We define that $\phi$ *depends* on $\Phi \subset \mathcal{L}_{\mathsf{Tr}}$ iff for all $\Psi_1, \Psi_2 \subset \mathcal{L}_{\mathsf{Tr}}$, we have that if $\mathsf{Val}_{\Psi_1}(\phi) \neq \mathsf{Val}_{\Psi_2}(\phi)$ then $\Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$.

Then Leitgeb shows that $D_\phi \stackrel{\text{def}}{=} \{\Phi \subset \mathcal{L}_{\mathsf{Tr}} | \phi \text{ depends on } \Phi\}$ is a filter.

Similarly $D^{-1}(\Phi) \stackrel{\text{def}}{=} \{\phi \in \mathcal{L}_{\mathsf{Tr}} | \phi \text{ depends on } \Phi\}$. Leitgeb shows $D^{-1}$ to be monotonic.

We define an ordinal sequence $(\Phi_\alpha)_{\alpha \in \mathsf{On}}$ as follows:

- $\Phi_0 = \emptyset$,
- $\Phi_{\alpha+1} = D^{-1}(\Phi_\alpha)$ and
- $\Phi_\beta = \bigcup_{\alpha<\beta} \Phi_\alpha$.

Least fixed point $\Phi_{\mathsf{lf}}$ of *grounded* sentences.

# Cantini

$\mathrm{Val}_\Psi(\phi)$ represents the truth value of the formula $\phi$ given that the Tr–predicate's extension is $\Psi$.
A set $\Psi \subset \mathcal{L}_{\mathrm{Tr}}$ will be considered *consistent* if, whenever $\psi \in \Psi$, then $\neg\psi \notin \Psi$.
An operator is defined as, for all $\Phi \subset \mathcal{L}_{\mathrm{Tr}}$,
$\mathrm{FV}(\Phi) \stackrel{\mathrm{def}}{=} \{\phi \in \mathcal{L}_{\mathrm{Tr}} | \forall\Psi \supset \Phi, \text{ s.t. } \Psi \text{ is consistent}, \mathrm{Val}_\Psi(\phi) = 1\}$,
Monotonous and consistency–preserving.
A sequence $(E'_\alpha)_{\alpha\in\mathrm{On}}$ is defined:

- $E'_0 = \emptyset$,
- $E'_{\alpha+1} = \mathrm{FV}(E'_\alpha)$ and
- $E'_\beta = \bigcup_{\alpha<\beta} E'_\alpha$. Its least fixed point is called $E'_\infty$.

# Conditional dependence formally (def. in Leitgeb[2005])

## Conditional dependence

$\phi \, \mathrm{dep}_\Sigma(\Phi) \stackrel{\text{def}}{=}$ for all $\Psi_1, \Psi_2 \subset \mathcal{L}_{\mathrm{Tr}}$ s.t. $\Sigma \subset \Psi_1, \Psi_2$ it holds that $\mathrm{Val}_{\Psi_1}(\phi) \neq \mathrm{Val}_{\Psi_2}(\phi) \rightarrow \Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$

- $\Phi_0^{AT} = \emptyset$,
  $\Gamma_0^{AT} = \emptyset$,
- $\Phi_{\alpha+1}^{AT} = D^{-1}_{\Gamma_\alpha^{AT}}(\Phi_\alpha^{AT})$,
  $\Gamma_{\alpha+1}^{AT} = \{\phi \in \Phi_{\alpha+1}^{AT} | \mathrm{Val}_{\Gamma_\alpha^{AT}}(\phi) = 1\}$,
- $\Phi_\beta^{AT} = \bigcup_{\alpha<\beta} \Phi_\alpha^{AT}$,
  $\Gamma_\beta^{AT} = \bigcup_{\alpha<\beta} \Gamma_\alpha^{AT}$,

Using that for all $\Phi, \Phi', \Sigma, \Sigma' \subset \mathcal{L}_{\mathrm{Tr}}$, for all $\alpha, \beta \in \mathrm{On}$,

1. If $\Phi \subset \Phi'$ and $\Sigma \subset \Sigma'$ then $D^{-1}_\Sigma(\Phi) \subset D^{-1}_{\Sigma'}(\Phi')$
2. (a) $\Phi_\alpha^{AT} \subset \Phi_{\alpha+1}^{AT}$ and (b) $\Gamma_\alpha^{AT} \subset \Gamma_{\alpha+1}^{AT}$

So a least fixed point, called $\Phi_{\mathrm{lf}}^{AT}$.

# Conditional c–dependence formally

## Conditional c–dependence

$\phi \; \mathrm{cdep}_{\Sigma}(\Phi) \overset{\text{def}}{=}$ for all *consistent* $\Psi_1, \Psi_2 \supset \Sigma : \mathsf{Val}_{\Psi_1}(\phi) \neq \mathsf{Val}_{\Psi_2}(\phi) \rightarrow \Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$.

- $\Phi_0^{\mathrm{c,AT}} = \emptyset$,
  $\Gamma_0^{\mathrm{c,AT}} = \emptyset$,
- $\Phi_{\alpha+1}^{\mathrm{c,AT}} = \mathsf{D}^{-1}_{\mathrm{c},\Gamma_\alpha^{\mathrm{c,AT}}}(\Phi_\alpha^{\mathrm{c,AT}})$,
  $\Gamma_{\alpha+1}^{\mathrm{c,AT}} = \{\phi \in \Phi_{\alpha+1}^{\mathrm{c,AT}} | \mathsf{Val}_{\Gamma_\alpha^{\mathrm{c,AT}}}(\phi) = 1\}$,
- $\Phi_\beta^{\mathrm{c,AT}} = \bigcup_{\alpha<\beta} \Phi_\alpha^{\mathrm{c,AT}}$,
  $\Gamma_\beta^{\mathrm{c,AT}} = \bigcup_{\alpha<\beta} \Gamma_\alpha^{\mathrm{c,AT}}$

# Reconciliation proof overview

- For all $\alpha \in \mathsf{On}$, $\Phi_\alpha^{\mathrm{c,AT}} = \pm\Gamma_\alpha^{\mathrm{c,AT}}$
- Redefinition
  - $\Gamma_0^{\mathrm{c,AT}} = \emptyset$,
  - $\Gamma_{\alpha+1}^{\mathrm{c,AT}} = \{\phi \in \mathsf{D}^{-1}_{\mathrm{c},\Gamma_\alpha^{\mathrm{c,AT}}}(\pm\Gamma_\alpha^{\mathrm{c,AT}}) | \mathsf{Val}_{\Gamma_\alpha^{\mathrm{c,AT}}}(\phi) = 1\} \stackrel{\mathsf{def}}{=} \Delta_c(\Gamma_\alpha^{\mathrm{c,AT}})$,
  - $\Gamma_\beta^{\mathrm{c,AT}} = \bigcup_{\alpha<\beta} \Gamma_\alpha^{\mathrm{c,AT}}$.
- $\phi \operatorname{cdep}_\Phi(\pm\Phi) \leftrightarrow \phi \in \pm\mathsf{FV}(\Phi)$
- For any consistent $\Phi \subset \mathcal{L}_{\mathsf{Tr}}$, $\Delta_c(\Phi) = \mathsf{FV}(\Phi)$
- For all $\alpha \in \mathsf{On}$, $\Phi_\alpha^{\mathrm{c,AT}} = \pm E'_\alpha$ and $\Gamma_\alpha^{\mathrm{c,AT}} = E'_\alpha$.